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ABSTRACT

One of a series of experimental units for children at the preschool level, this booklet deals with geometric concepts. A unit on volume and a unit on linear measurement are covered; for each unit a discussion of mathematical objectives, a list of materials needed, and a sequence of learning activities are provided. Directions are specified for the teacher. Appendices include student worksheets and patterns for geometric models. For other documents in this series, see SE 016 124 through SE 016 129. (DT)

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Practical Paper No. 27

GEOMETRY

Michael L. Mahaffey

August, 1969

Research and Development Center in Educational Stimulation

University of Georgia Athens, Georgia

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Note to the Teacher

This is an experimental unit dealing with geometric concepts which we hope to develop through an intuitive approach, rather than the formal connotation normally associated with the topic. In this booklet, you will find a discussion of the mathematical objective of the unit and a sequence of learning activities designed to help children grasp the concepts outlined. The time required for this unit will depend on the previous experiences of the children and school time alloted for its study.

It is the author's intent to make the activities of this unit so naturally pleasant for children that they will engage in them without outside compulsion. If you find that you have to force a child to take part in these activities, it would be better to let that child do something else for a time. Children should not, however, be excused from participation on the basis of a passing whim or momentary attraction to something else. Please try to make the suggested activities gamelike or playful in spirit. It is, after all, not essential for a child to do these things, although we feel it is desirable.

One question deserves a rather extended explanation. Throughout this unit, children will be encouraged to do a variety of tasks, some more difficult and some less difficult. Every child will experience failures and successes. In total, how we respond to a child's failures and successes will have an impact on his ultimate attitude toward school, teachers, and learning. Of course, a correct answer deserves a rewarding response, one which indicates that you are pleased with the child and happy that he has succeeded. If a child does a task incorrectly, how shall we respond? Try not to say "no",

or "that is wrong." Instead, go back to the learning activity and ask yourself, "How was the child to get the answer to that question?" By recalling the learning activity, try to get the child to see how he could have found the correct answer and why his answer was wrong. In this way, you are a guide to thinking processes and not a judge of the product. This is an experimental unit. If it goes well, we are all happy. If it goes badly, it is because the unit needs improvement and not because of you. Please let us know if you find difficulty in using this material. We stand ready to help you change or supplement the material, or to withdraw it if it is not working.

Michael L. Mahaffey

UNIT I

VOLUME

Purpose:

The major purpose of Unit I is to give the child experience with the concept of volume--"How much does a three-dimensional object hold?" In broad terms, this encompasses the following:

1. Giving the child experience with handling different types and shapes of three-dimensional objects.
2. Giving the child experiences which show that a three-dimensional object holds something, that it is a container.
3. Helping the child develop an intuitive feel of how much or how many a container will hold. Incorporated within this is estimation.
4. Giving the child experiences with different objects, which, due to different size, shape or some other physical characteristic have different capacity.
5. Giving the child experience in ordering three-dimensional objects by volume, and hence, with the transitive property of the relations holds more than, holds less than, and holds the same as.
6. Helping the child discover the need for agreement on how and with what we are to measure volume (this with no formal introduction or instruction in standard units).
7. Giving the child experiences which show conservation of volume.
8. Giving the child further experience in counting.
9. Giving the child experience in classifying three-dimensional objects by the shape of faces, i.e., square, triangular, etc.

Materials:

In general the same materials will be used in each of the various learning activities. No attempt is made to exhaust the possibilities, but the following

could be regarded as the minimum:

Measuring devices

1. Various solid spherical objects such as large marbles of the same size, ping-pong or Styrofoam balls, baseballs, volley balls, etc. Many of the smaller devices will be required.
2. Similarly, various sizes of children's wooden or plastic blocks, triangular solids, rectilinear solids, etc.
3. Sandbox with sand--preferably a small box for each table.
4. Water (if teacher deems it advisable).

Containers to be measured

1. Many cardboard boxes of various sizes such as cut-off cereal boxes, cigar boxes, shoe boxes, etc; cut-off milk cartons to contain half-pint, pint, quart and half-gallon.
2. Cylindrical containers of various sizes such as coke cups, coffee cans, jars, cones, etc.; also small measuring cups or similar scoops.
3. Odd-shaped plastic containers obtained by cutting the tops from shampoo bottles, detergent bottles, Clorox bottles, etc.

ACTIVITY SEQUENCE I

Objectives:

1. To motivate further interest in geometry.
2. To provide such sequencing of activities as will enable the child naturally to become familiar with dimensional objects.
3. To give the child an intuitive concept of volume (contains or holds something).

Activity 1:

To give the children experiences with different types of three-dimensional objects, and to help them discover what attributes a three-dimensional object has before learning its name.

Arrangement:

Divide class into groups of 3-6 children per table.

Materials:

On each table place a wide variety of three-dimensional objects. In particular, insure that there are different sizes of each kind of material which is to be used in the entire unit. (Don't put balls on one table, blocks on another, etc.)

Procedure:

1. Give the children a few minutes of free play. Teacher can visit each table and make a kind of game of describing the various objects in as many ways as possible.
2. Now, the teacher shifts the emphasis in the game. She goes from table to table, selects an object. and sees if any child at any particular table

can properly identify it. After it has been named correctly, she holds it up for the class to see and has the class repeat the name in unison. Teacher must insure that a large cube or a small cube is still recognized as a cube. Also, she will want to make a list for herself of the objects that she feels the children should be able to name when the class later discusses different items. Remember that correct names for objects are as easily learned as incorrect ones.

Activity 2:

To give children experiences in manipulating three-dimensional objects to illustrate that they can hold something, that they are containers.

Arrangement:

Children remain at their tables as for Activity 1.

Materials:

On each of the pupils' tables there should be various measuring devices as well as various small containers. In particular have some small balls, marbles, blocks, small boxes, jars, coffee cans, and milk cartons on each table. In front of the class where all can see, the teacher should have boxes larger than those on the children's tables, large cans, wastepaper basket, volley ball, baseball, stack of books, etc.

Procedure:

1. Teacher is to ask a series of questions, letting the children answer by actual selection of some three-dimensional objects arranged in front of the class:

   (a) "Find me a container on your table that I can use to carry sand to the play yard." Insure that each child selects an open container,

not a solid one.

(b) Hold up a baseball and ask the children to find on their table a container in which they can carry the ball to the play yard. Not all children will be able to have different objects due to limited amount of material on each table.

(c) Clear all containers from the children's tables. The teacher chooses a stack of books to be carried to another room. Then direct the attention of the class to the larger containers near the teacher's desk and invite the children to choose a container suitable for the books. It is hoped that a square box will be selected, not the wastepaper basket.

(d) Teacher should continue to set up similar situations and ask questions, forcing the children to focus attention on the holding capacity of the various containers.

2. With children working in small groups at a table or desk, have them experiment to find which container will hold the different kinds of objects. (This can be an individual project.)

Activity 3:

To relate the holding ability of a container to its size, and to begin to introduce the notion of estimation and how it can be discovered whether an estimate is correct or close.

Arrangement:

Children remain at their tables as before.

Materials:

Same as Activity 2, except teacher will need each of the various objects and containers on or near her desk.

Procedure:

1. Teacher involves entire class by holding objects so that all can see, and by asking questions.
   (a) Hold up a baseball and half-gallon milk carton and ask, "Will this container hold the ball?" After the children have responded, let one come up and try. Do this with several objects and containers, letting object drop into the container each time.
   (b) Choose a container that is just a little too small to hold the ball and repeat the experiment, each time making sure that some child has a chance to verify the estimate.
   (c) Have two containers which seem to be of the same size but are such that one holds the ball and the other does not. Allow several children to experiment with fitting the ball into the container while questions are asked as to why one holds the ball and the other does not.
   (d) Now, at a little more abstract level, hold up a pint milk container and ask: "Will this hold a marble?" "Will it hold a baseball?" Do not show the object, i.e., the marble or the ball.
2. Teacher visits each table and chooses an object. She then requests the children at that table to pick a container which will hold the chosen object. Be sure to let each child try his container to see if he is right or wrong.
3. The teacher holds up various sizes of objects so that all may see and asks children to select containers from their table which will hold them. Again, be sure to include among the objects one which no container will hold and try to elicit why from the children.

ACTIVITY SEQUENCE II

Introduction:

The shift from Sequence I to Sequence II should be gradual and natural. After becoming familiar with the idea that a three-dimensional object can hold something, the children should be ready to begin determining how much or how many of certain objects a container will hold. Whenever possible, preface each activity by having the students estimate and, if appropriate, record their estimates. In a real sense the activities will be the checking of their estimates by the children.

Objectives:

1. To give the students experiences leading toward the conclusion that two containers of the same size hold more of one kind of object than they do of another, and that this difference is due to the size (possibly shape) of the measuring material.
2. To give the students experiences leading toward the conclusion that different-sized containers hold different amounts of objects or materials of the same size.
3. To give the students experiences leading toward the conclusion that sand is a better measuring medium than marbles or balls.

Activity 1:

Measuring with the same-sized container but different-sized measuring devices.

Materials:

A wide variety of measuring materials such as cubes, marbles, balls, and sand; and a variety of containers such as small boxes, milk cartons, jars, coffee cans, etc. at each table.

Arrangement:

In groups at tables as before.

Procedure:

1. The teacher selects from each table a container and has the children estimate how many marbles or small balls it will hold.
   (a) Record the estimates, and then let students fill the containers, counting and recording as they fill.
   (b) These activities should be followed by a class discussion as to why the containers hold more marbles than Ping-Pong balls.
   NOTE: If children have not reached the stage of counting and recording, then select two containers of the same size and shape, and have children fill them alternately, one marble then one ball. The idea of "one-one ness" should bring out that more marbles than balls are needed.
2. To lead into more abstract thinking, the teacher should follow Activities 1 and 2 with questions such as, "Will this can hold more peas or base-balls?" The teacher can use any well-known objects as long as they are in sight of the students.
3. At some stage in this activity, the teacher selects two different containers of the same size and repeats the above experiments. Again, use different-sized measuring devices in an attempt to bring out that we need a uniform measuring device.

Activity 2:

Measuring with the same measuring device as in Activity 1, but using different-sized containers.

Materials:

As in Activity 1, but the use of sand as a measuring device is especially recommended during later stages of activity.

Arrangement:

As in Activity 1.

Procedure:

1. The teacher selects two containers of different sizes for each table. Elicit from the children that the sizes are different. Let the children fill the containers with marbles, sand, etc. to check that the two are not the same size. Then have a class discussion as to why different numbers of balls, cupsful of sand, etc. are needed to fill the containers.
2. Repeat the above with various types of measuring devices and containers but at some stage be sure to shift to sand as a measuring device and glass jars as containers. Try to elicit from the students why the sand made a better measuring medium than marbles, balls, etc.
3. The teacher selects jars or containers that students can see through. Lead discussion as before, "How many cupsful of sand will this one hold?" Let students fill the containers to check their estimates, and have them record their findings. In doing this, use at least two different-sized measuring cups to fill the containers. Through class discussion, again bring out that sand <u>filled</u> better than marbles, etc., but that the class is still faced with the problem of having to choose measuring cups of the same size.

ACTIVITY SEQUENCE III

Introduction:

This is a continuation of Sequence II introducing a few more subtle ideas for the children to grasp. It seems only natural to start comparing three-dimensional objects on the basis of how much or how many they hold. Without formally introducing the word dimension, it is hoped that the idea becomes clear that capacity is changed if the height or base size is changed. Also, children should be aware that there are objects which have no holding capacity.

Objectives:

1. To give the student experiences in classifying three-dimensional objects according to their holding capacity.
2. To learn to order containers according to how much they hold.

Activity 1:

Learning the concepts of bigger and smaller as related to volume.

Materials:

As in Sequence II.

Arrangement:

As in Sequence II.

Procedure:

1. Before class, the teacher selects two milk cartons of the same capacity; i.e., both may be pints, but the tops should be cut off so

that one will hold more than the other. Do this for each table.

(a) Have children estimate how many cupsful of sand each could hold, record the estimates, and then check by filling.

(b) Through class discussion, elicit that we can say that one container is bigger or smaller according to how much sand they hold.

2. Repeat the above, using different-shaped containers, e.g., a detergent bottle and a coffee can. Be sure to let the students examine the containers first, and then check their estimates by filling with sand.

Activity 2:

Ordering by volume.

Materials:

As above.

Arrangement:

As above.

Procedure:

1. Using three different-sized containers such as milk cartons, coffee cans, and jars, have the students fill the containers and arrange them according to which holds the most sand. Be sure these containers can be differentiated, i.e., A, B, or C; or red, yellow, blue. The teacher must not identify them by size. Now class discussion can be started with questions such as: (a) "Does B hold more than A?" "Why?" (b) "Does C hold more than B?" "Why?" (c) "What do we know about A and C?" "Why?" It is not expected that the transitive property will be noted, and no attempt should be made to force it.

2. After repeating the above with various materials, containers, and in

various ways, the children fill preselected containers of the same size, and the teacher asks them to arrange them as in earlier lessons. It is hoped that the idea of same size as will come forth just as bigger than etc. did before.

3. The teacher should now place from three to five different-sized containers on each table and let the children estimate their volume and arrange them according to size. Record this arrangement, and then let the children fill the containers with sand to check their estimates.

Activity 3:

Experiments in varying dimensions.

Materials:

Models similar to attached sheet.

Arrangement:

At tables as for previous activities.

Procedure:

1. Using models, follow activity prescribed on attached sheet.

Prepare containers of at least four different heights but all having the same base as in Figure 1 below. Have the children fill the containers with various materials, marbles or sand, and see if the idea occurs to them that the volume change is due to the change in height. Check also for the idea that certain objects can hold nothing.

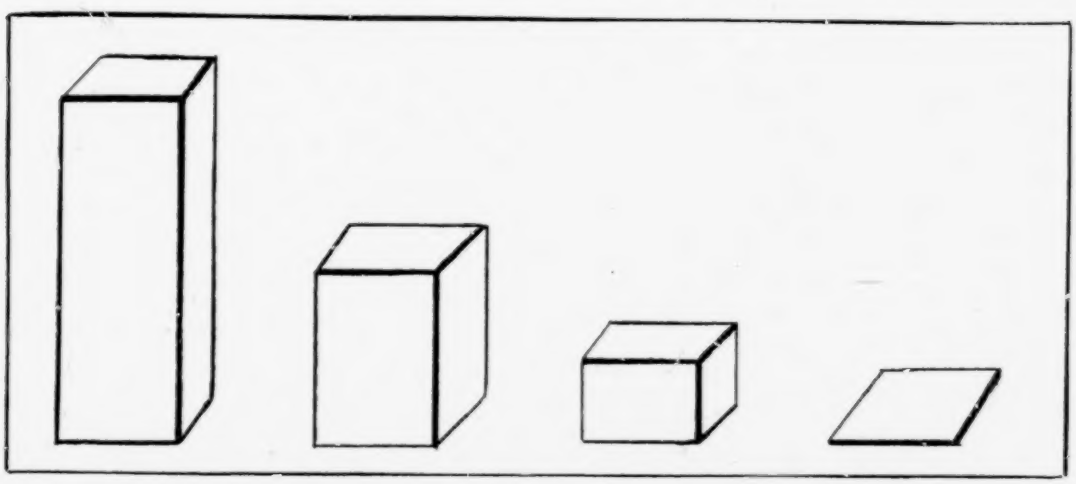


Figure 1

Prepare a second set of containers, letting the height remain constant while the base varies. Again, engage the class in activities until change in volume is associated with the change in base.

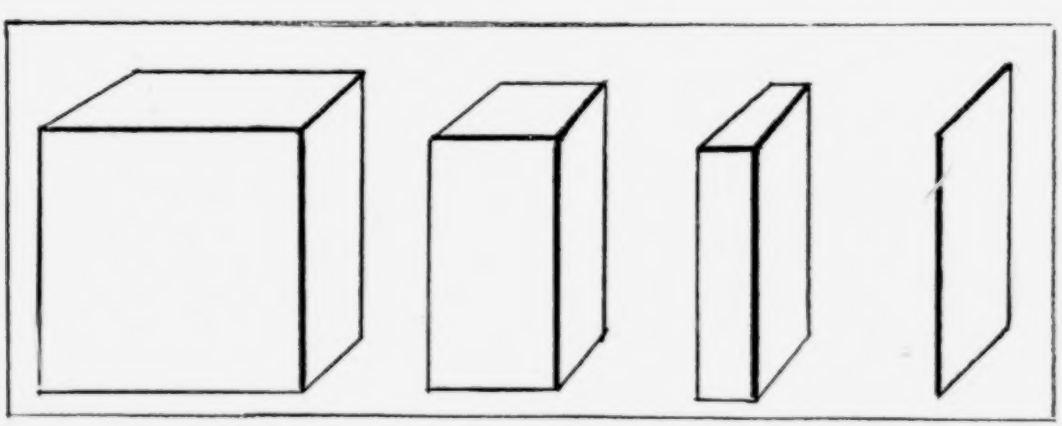


Figure 2

UNIT II

LINEAR MEASURE

Purpose:

The major purpose of Unit 2 is to give the child experience with linear measurement so that he may grasp the concept that "measurement is the comparison of an object with a given unit of measure." To reach this goal, however, the following will be necessary:

1. To give the child experience with using a straight-edge.
2. To further the concept of ordinality of the natural numbers.
3. To encourage the child to observe and compare geometric figures.
4. To develop the child's ability to verbalize his observation.
5. To introduce the child informally to the vocabulary necessary for communicating geometric ideas.
6. To deepen the child's understanding of conservation of length.
7. To provide the child with an intuitive understanding (feeling) for the arbitrariness of the names of units of measure.
8. To further develop the child's classification skills.
9. To develop in the child an understanding of and a need for standard units of measure.
10. To encourage a child to observe patterns and to make generalizations.
11. To make the learning of mathematics both meaningful and enjoyable.

Materials:

The materials for this unit are such that a great many are available within the average classroom. However, some must be obtained from other sources and this should be done well in advance so that every child has the necessary equipment.

The following is a list of the more meaningful items needed:

A 1-foot piece of lattice (or molding) for each child.

A 1-yard piece of lattice (or molding) for each child.

A 1-inch piece of lattice (or molding) for each child.

A 6-inch piece of lattice (or molding) for each child.

Geoboards and rubber bands; yarn; pegboard (2ft x 2 ft., with 3/16" x 1" pegs); clothes pins; kite string; clothes hangers.

It is believed that the physical skills of 6-year-olds are such that we can not automatically assume that they are able to master the use of a straight-edge. A minimal amount of observation of the attempts of this age group at this activity will immediately bear out this assumption. For this reason the first few exercises concentrate, with little exception, on the development of such a skill.

ACTIVITY SEQUENCE I

Objectives:

1. To introduce the straight-edge as a tool for constructing a line segment between two points.
2. To provide an experience in which ordinality of the natural numbers may be utilized.
3. To provide for an intuitive understanding of point.

Activity 1:

For this activity the children should be asked to bring to school pictures of animals, which they may display on the bulletin board.

Materials:

Felt board and felt cutouts.

Arrangement:

Class works as a group.

Procedure:

1. Have each child show his pictures to the class and tell what the picture shows. The teacher may want to display the pictures on the board. As each child tells about what he has brought, it should be stressed that these are pictures of animals and not the actual animals. Based on this activity, we want to introduce the word <u>represents</u>. We say, the picture represents the animal.
2. A similar procedure should be used with the felt cutouts.

Activity 2:

This activity lends itself very well to class discussion and the teacher should encourage as many pupils as possible to participate. Also, the natural use of ordinal numbers can be utilized.

Materials:

Student coloring book (Appendix I)

Arrangement:

Children work at their tables or desks.

Procedure:

1. Give each child a coloring book. Explain that we have forgotten to number the pages and ask each child to help do this. Be sure he numbers correctly.
2. Discuss how books are numbered and why we don't number the cover.

Activity 3:

Now that the pages are numbered each child can find the right place in the book.

Materials:

Coloring book (Appendix I), pencil, crayons.

Arrangement:

Individual work at pupils' desks or tables.

Procedure:

Have the children turn to page 1 of their books and ask them to draw the

lines from point to point, starting with the point labeled '1' and then proceeding to connect the rest of the points in order. Most children have already been introduced to this type of activity, however, there may still be a need to give an example at the board. (Be sure the children do not use straight-edges.)

Activity 4:

Discuss with the children the fact that their lines were not always straight. Ask them if they know of any way they could make them straighter. The students will most likely make some recommendations such as the use of the edge of a book.

Materials:

Straight-edge (not a ruler), coloring book (Appendix I).

Arrangement:

Individual desk work.

Procedure:

1. Give each child a straight-edge (6" piece of lattice or molding, not a ruler). Turn to page 2 of the coloring book and use the first three or four frames as examples of how to draw a straight line between two points. (You may wish to have the children number the frames as they did the pages of their books.)
2. Now, let each child complete the remaining frames by himself, with as much individual help as needed. Try to help the children acquire a functional technique for using the straight-edge. In giving instructions, you could say something similar to: "Draw the line between the two points." or "Draw a line from one point to the next."

Activity 5:

In this activity each child will work on his own coloring book. Again class disucssion should be elicited.

Materials:

Coloring book (Appendix I), crayons.

Arrangement:

Individual work.

Procedure:

1. Have the children complete the picture on page 3 of their coloring book by drawing the lines from dot to dot. Once they have completed the figure have them color the largest fish red and the smallest fish blue. Upon completion of this task, ask, "Which fish is inside the bowl?" "Which fish is outside the bowl?" This activity is to provide the child with an intuitive encounter with the words inside and outside, each of which will be used later in a more mathematical setting.

Activity 6:

Use pages 4-6 in the coloring book to give the children further practice with the straight-edge. Provide help as needed, and be conscious not only of the line-drawing ability of the child, but also of the way he is ordering the numbers.

ACTIVITY SEQUENCE II

Objectives:

1. To introduce comparison activities by discussing triangles, quadrilaterals, and other polygons.
2. To continue the development of skills in using the straight-edge.

Activity 1:

In this activity the burden of discussion should be left to the students. The teacher's role is that of a moderator.

Materials:

Coloring book (Appendix I), straight-edge.

Arrangement:

Class works as a group.

Procedure:

Have the children complete the figure on page 7 of their coloring book by using their straight-edges. Once they have completed this task, follow up by asking questions such as the following: "Are all the figures the same?" "How are they the same?" "How many sides does each have?" "Can you draw a picture of a point inside the figure?" "Can you draw a picture of a point outside the figure?" "Draw a picture of a point on the figure."

Activity 3:

This activity will call for discussion similar to that used in the two previous activities. Each comment by the children should be discussed by the

teacher in a positive manner whether it is right or wrong. The incorrect answer may be as instructive as a correct one. Our primary concern in the introduction is that of comparison by number of sides. Also, while these figures are quadrilaterals, we should not necessarily use the word at this time.

Materials:

Coloring Book (Appendix I), straight edge.

Arrangement:

Class works as a group.

Procedure:

Have the children complete pages 8 and 9 of their coloring book, using a straight-edge to draw the line segments between the dots. Upon completion, the questions used in the previous two activities should be used again.

Activity 4:

This activity is to be worked by each child individually. A group discussion of the task may follow if needed. The primary interest is in making the numerical distinction between polygons. (You may use the word polygon but do not require the children to do so.)

Materials:

Coloring book (Appendix I).

Arrangement:

Individual desk work.

Procedure:

1. Have the children turn to page 10 of their coloring book. After a brief introduction, ask the children to:

   (a) Place an (x) inside each four sided polygon,

   (b) Place a (✔) inside each three sided polygon,

   (c) Place an (*) inside each five-sided polygon.

2. Check the page for accuracy. You will want to talk about the circle and how many sides it has. (None, since a side must be a straight line.)

ACTIVITY SEQUENCE III

Objectives:

1. To introduce the geoboard.
2. To provide readiness activities for future work with congruence.
3. To provide further experiences in which the child may develop his verbal skills and the art of observation.

Activity 1:

In this activity we expect that the children will make many different geometric shapes on their geoboards. Also, it is natural for them to want to share what they have made (created) with other children. At first, this may take the form of showing their neighbor. This should be encouraged as long as the class is not too chaotic. Ultimately this need to share can be channeled into class discussions.

Materials:

Geoboard, rubber bands.

Arrangement:

The class may be divided into small groups after the initial introduction of the apparatus.

Procedure:

Provide each child with a geoboard and three rubber bands. Once the children have the geoboard, allow them a reasonable amount of free time to create any kind of picture they wish by encircling pegs with rubber bands.

As this free play continues, it will lend itself to discussion of their pictures. Let the children describe the pictures as they see fit; there is no reason to force mathematical description of these endeavors. This exercise has no definite time schedule but is terminated as the teacher sees fit. Remember we only want to familiarize the pupil with the apparatus and to provide for free discussion of what he sees and produces.

Activity 2:

This activity will provide a prime opportunity to recall previous class discussion of polygons. The child must now draw upon his verbal understanding of past vocabulary. The use of the geoboard offers opportunity for a wide variety of shapes to be created and discussed by the children. They are no longer restricted to figures that the teacher makes up (Activity Sequence II).

Materials:

Geoboard, rubber bands.

Arrangement:

Class arrangement is left to the discretion of the teacher.

Procedure:

Each child will use his own geoboard for this activity. The goal is to have the children follow simple oral directions. Ask each child to make a triangle with a rubber band on his geoboard. It should be noted at this time that all the triangles are not the same (congruent), but that they all have three sides (see Figure 3). A similar activity and discussion can and should be used for the rectangle and the square (quadrilaterals, Figure 4). The activity should be continued for figures with 4, 5, 6,..., sides so that the student learns there are geometric figures other than those with standard names. (Vocabulary is important only to the extent that the teacher uses it correctly.)

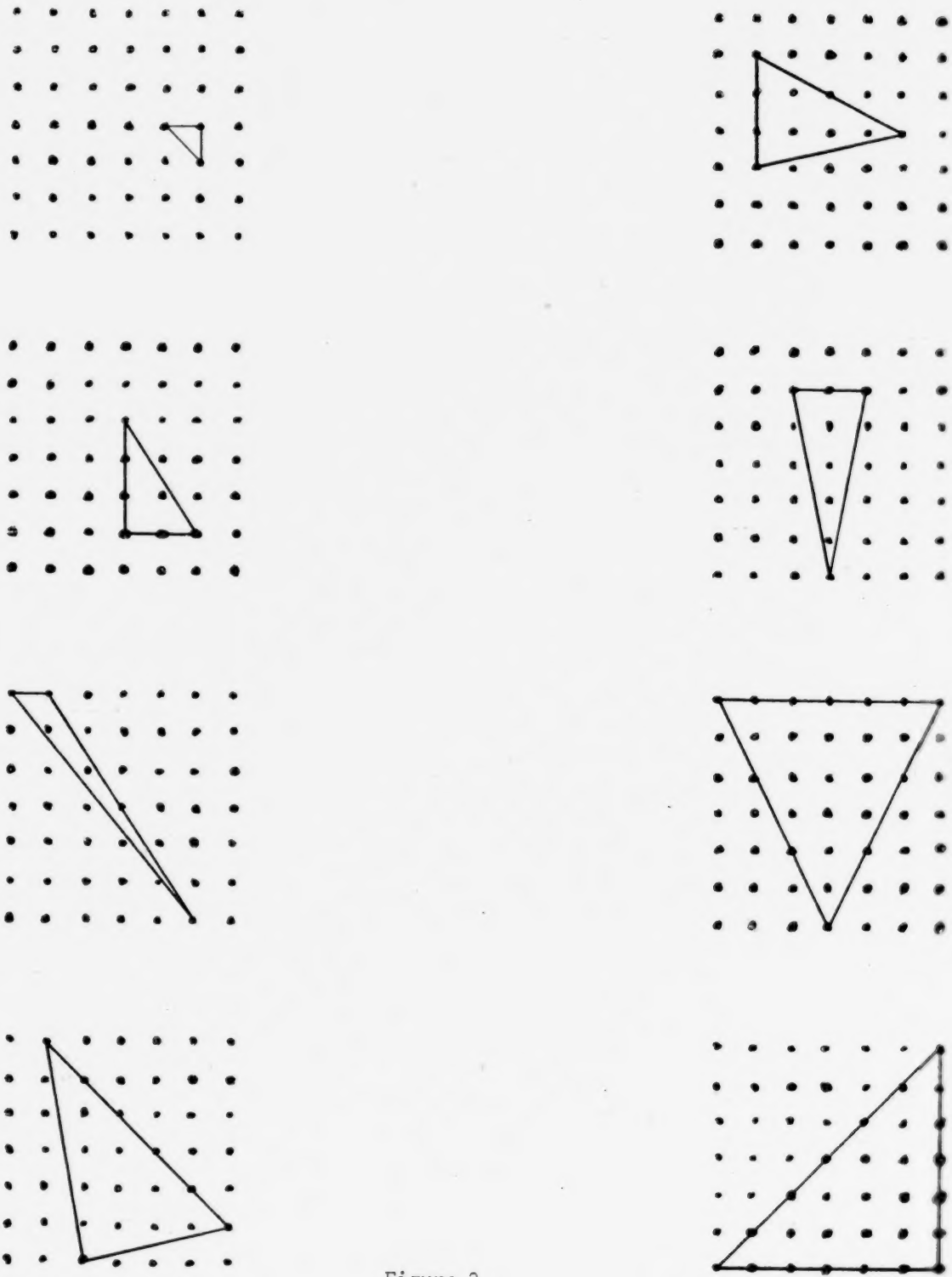


Figure 3

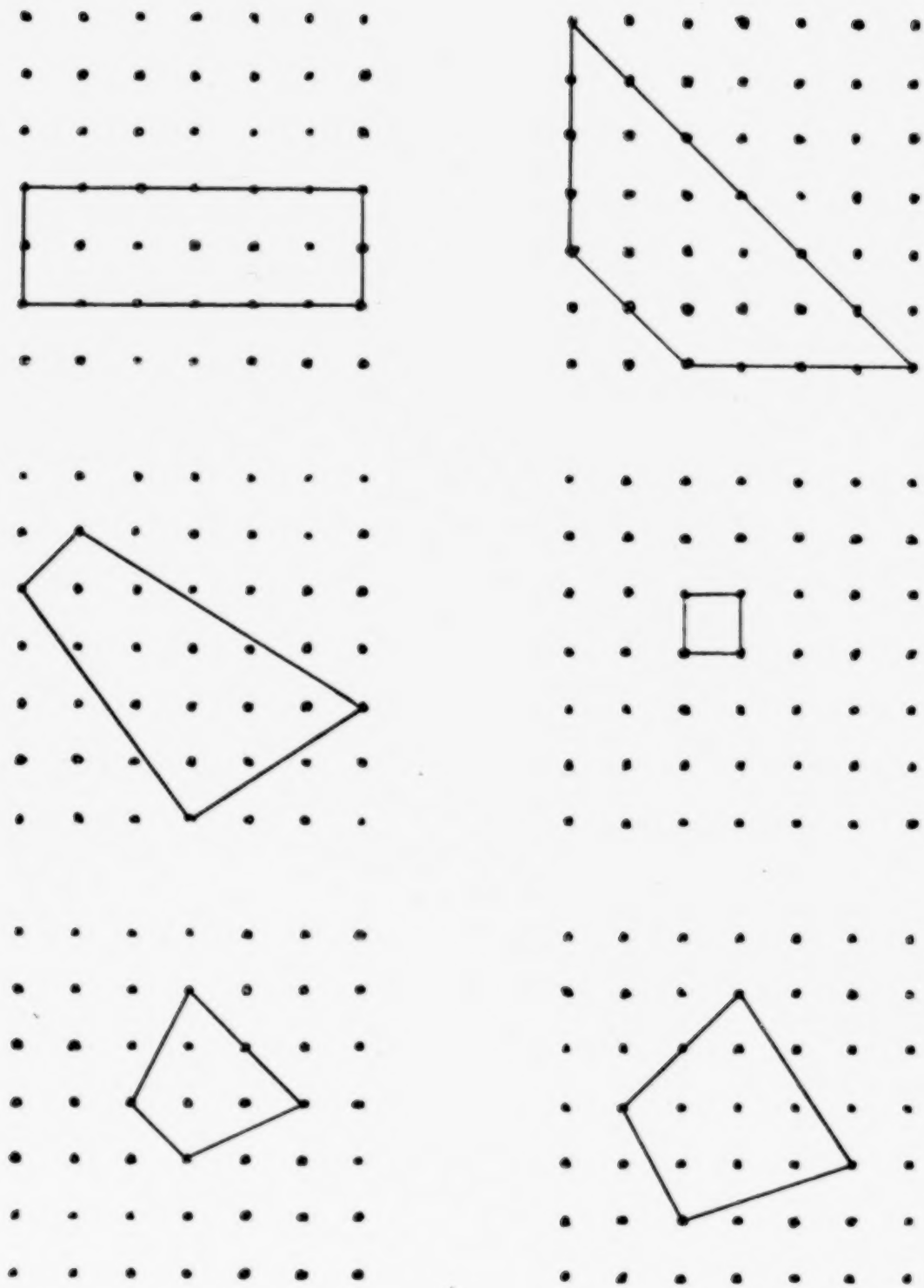


Figure 4

Activity 3:

We are interested in introducing the phrase copy exactly but we are not looking for proficiency in using it. The teacher should not feel that each child must perform the task correctly; this will only come with time and experience.

Materials:

Geoboard, rubber bands.

Arrangement:

Class as a group.

Procedure:

The teacher will make a square on her geoboard, and then ask the students to copy it exactly, i.e. The copy must be congruent to the figure the teacher has made. At this point, class discussion will focus on what we mean by copy it exactly. While we are looking for an intuitive understanding of congruence, some verbalization of its meaning will naturally appear in the class discussion. We need not discourage it, nor must we require it. This activity should be repeated with other figures such as the triangle, rectangle, or any of the other figures illustrated (Figure 5).

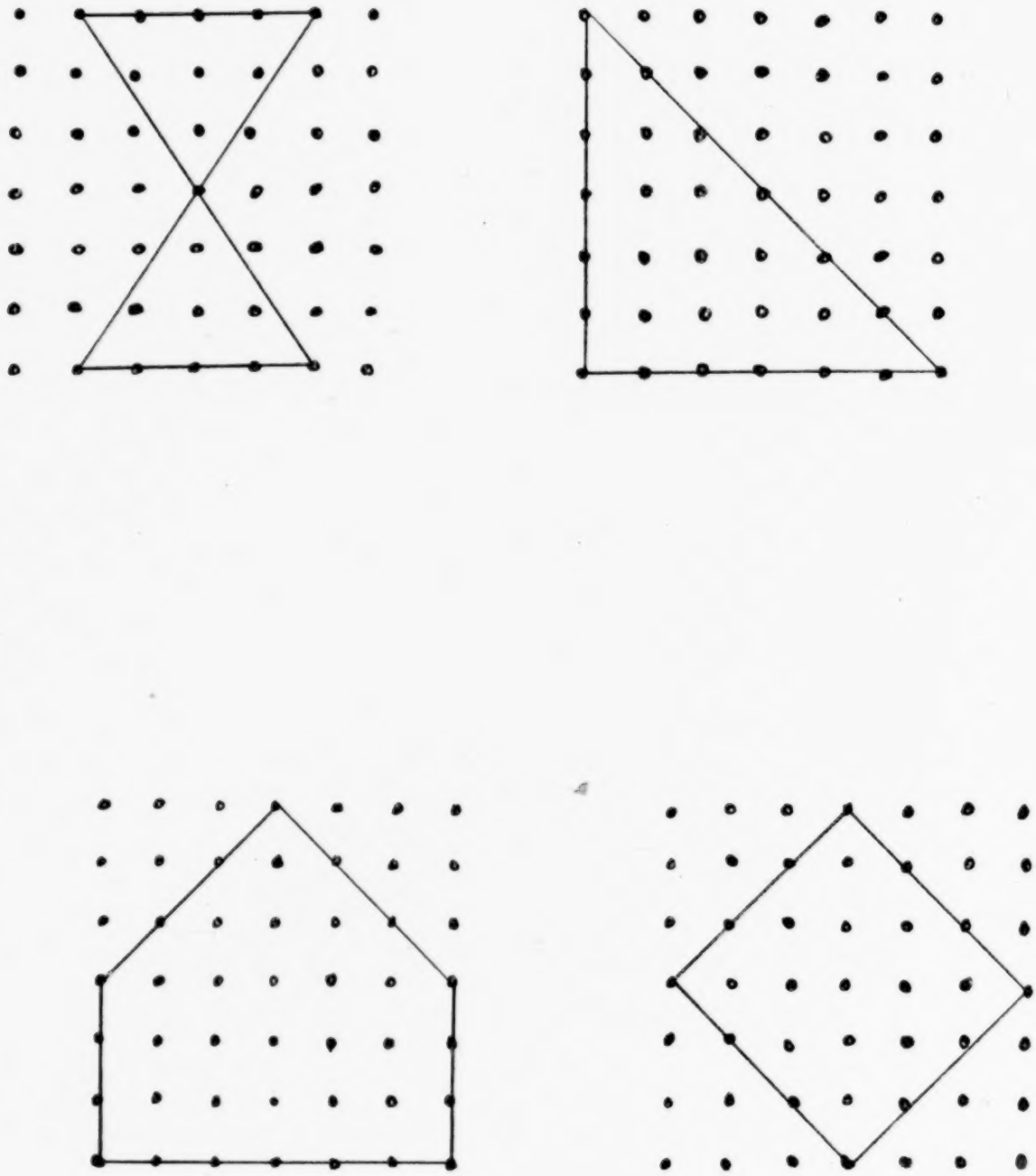


Figure 5

Activity 4:

This activity will help further the notion of congruence through actual physical manipulation of figures.

Materials:

Cardboard cutouts (Appendix II), chalk.

Arrangement:

Small groups.

Procedure:

Trace around one of the cardboard figures on the floor; then place all the figures on the desk. Now, ask one child to find the figure that fits inside the lines exactly. Once he finds it and selects a figure, he must check it by placing it inside the traced lines.

The first few times the figures should be placed face up, that is, the side of the paper on which the tracing was done facing upward. For example, if we traced the figure shown below and turned it over, it would not fit inside the

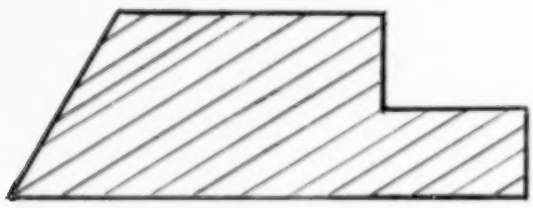


Figure 6

line. This type of activity will give the child an intuitive idea of checking congruence by superposition.

ACTIVITY SEQUENCE IV

Objectives:

1. To review the notion of a dot being a representation of a point.
2. To present an intuitive development of a line-segment as the shortest distance between two points.
3. To introduce the concept that a plane figure is made up of line segments.
4. To develop an intuitive understanding of inside, outside, and on a plane figure.
5. To reinforce the concept of set intersection through the simultaneous use of several different geometric figures and discuss the terms inside, outside, and on.

Activity 1:

This activity is an extension of Activity 1, Sequence I presented earlier. We are trying to introduce a new word into the child's vocabulary through use, without directly defining the word for him.

Materials:

Feltboard, felt cutouts.

Arrangement:

Class works as a whole.

Procedure:

Have a pupil pick out an animal cutout and place it on the feltboard, then, have him ask the class what it is (duck, rabbit, bird, etc.). Ask the class if

the thing on the board is a real animal, and then ask a child, "Since it's not real, what is it?" During this activity, the teacher should guide the children to the understanding of the phrase, "It represents the animal."

At a later date we will want to use the word represents when we refer to geometric figures (a dot represents a point). While this idea is desirable, we prefer not to belabor it. If it were too difficult for the children, we might lose their interest by pressing the idea.

Activity 2:

We are primarily interested in having the child associate the word path with a geometric example. He needs to think of a path as a means of getting from one place (point) to another. This can, of course, be facilitated by first using the term with reference to the child's activities in a phrase such as, "What path do we take to the playground?" or "Have you ever seen a path through the woods?"

Materials:

Ball of string, yarn, feltboard, felt cutouts.

Arrangement:

Small groups of 4-6 students working with the teacher. This arrangement is very desirable in that each child is allowed full opportunity to express himself.

Procedure:

1. Ask a pupil to place the felt cutout of the rabbit and the pear on the board. Direct another pupil to mark with yarn the path he thinks the

rabbit might take to get the pear (see Figure 7 below). You may wish to use other objects on the board to encourage the use of different paths.

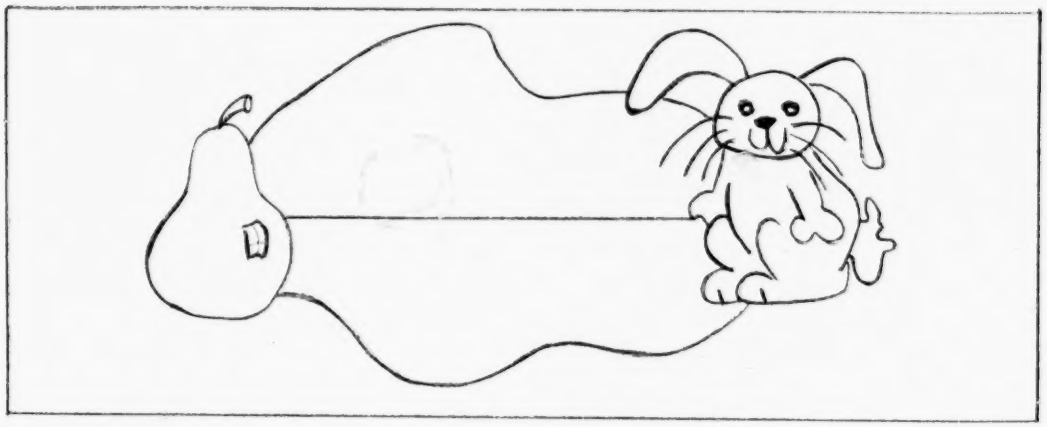


Figure 7

2. Using the ball of string, similar activities may be performed around the classroom. For example, mark different points in the room with tape or chalk, and ask a child to show a path from point to point. Continue this type of activity as long as interest holds or until the pupils comprehend the meaning of the term path.

Activity 3:

This activity is to give the child a basis for intuitive understanding that the shortest distance between two points is a straight line. The children may prefer to say the "quickest way between two places." This can be accepted at first and changed later.

Materials:

Feltboard, cutouts, yarn.

Arrangement:

Class works as a whole.

Procedure:

1. Place cutouts of a bird and a birdhouse on the feltboard. Take three different lengths of yarn, of which one should barely reach from bird to birdhouse, the other two lengths should be longer (see the drawing below).

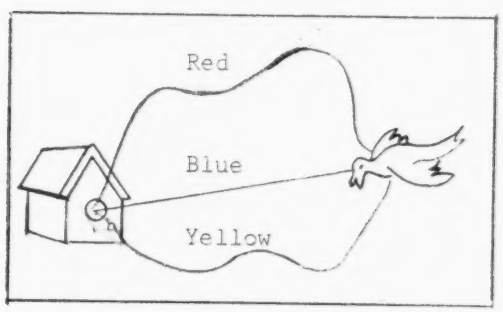


Figure 8

Have three pupils come to the feltboard and each use one piece of yarn to mark a path the bird could take to the house. Which piece of yarn shows the shortest path?

2. This activity is only an example and should, by no means, be the sole experience from which the shild is expected to learn. Present as many similar activities as are necessary.

Activity 4:

This activity is used to present the phrase line segment as a replacement for the shortest distance between two points is a straight line. The child should not be expected to memorize this latter definition; he is just to see as many instances as possible where it is used.

Materials:

2-ft. by 2-ft. pegboard, several 3/16" x 1" pegs, long rubberbands, cutouts of different objects (house, swing, boy, etc.).

Arrangement:

Class as a whole. Call on individuals to perform the different tasks in front of class. This work will also lend itself very well to small groups of perhaps 4-6 pupils.

Procedure:

To further the discussion of paths, set up situations with the cutouts on the pegboard (see the drawing below).

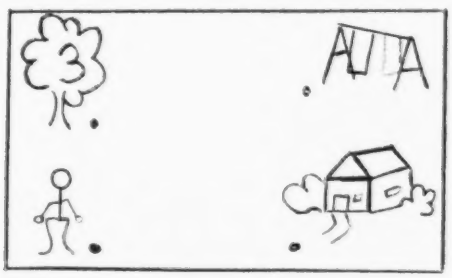


Figure 9

Place the pegboard in front of the class, and have the pupils identify what they see. Ask a student to use a rubberband to show a possible path the boy or girl could take to the swing set. Repeat until the students have displayed at least three different paths. Now ask the students which path is the shortest. After several similar examples, the children could be reminded that they were drawing line segments when they were using the coloring books.

Activity 5:

In this activity we will discuss inside, outside, and on a plane figure. This is the third and most formal presentation of these concepts. We can expect that most children will have a firm grasp of these ideas upon completing the following sequence of activities.

In addition, we wish to use the terms path, sides, and line segment to refer to the different figures that we construct. This is not to say that every child must use this vocabulary, but that the teacher should use it correctly to provide an informal example for the children. This effort to illustrate the correct use of inside, outside, on, and line segment with respect to plane figures is a good readiness activity for the concepts of area and perimeter to be discussed later.

Materials:

Pegboard, pegs (plain and colored), rubber bands (or any other material that will serve the same purpose).

Arrangement:

Use large or small group as appropriate.

Procedure:

1. Draw a square, rectangle, and circle on the chalkboard, have the pupils identify them. (The pupils may be asked to draw the figures for the class.)
2. Now, draw a large triangle on the chalkboard and trace with your finger

the path or line segments that form the figure, stressing that the line segments, not the interior, make up the triangle.

3. Draw a large square or rectangle and have a student come forward and trace the path that forms the figure. Have one of the students tell how many line segments (sides) make up the figure.
4. Construct a triangle on your pegboard with a large rubber band and three white pegs (see Figure 10). Then take a colored peg and place it first

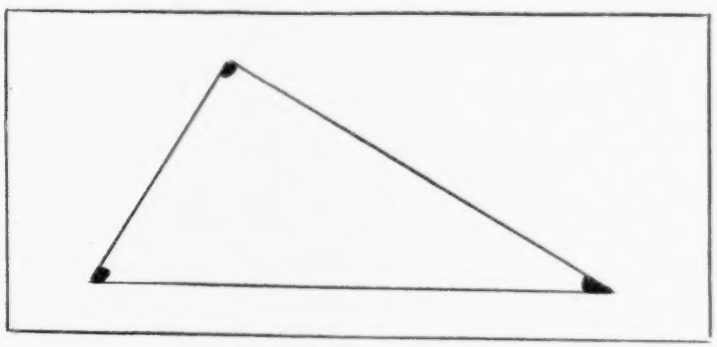


Figure 10

inside and then outside the triangle while a pupil tells what position the peg is in. (Pupils may be asked to place the peg.)

5. Use the above activity with a square and a rectangle.
6. Construct a square and a triangle on your pegboard (see Figure 11).

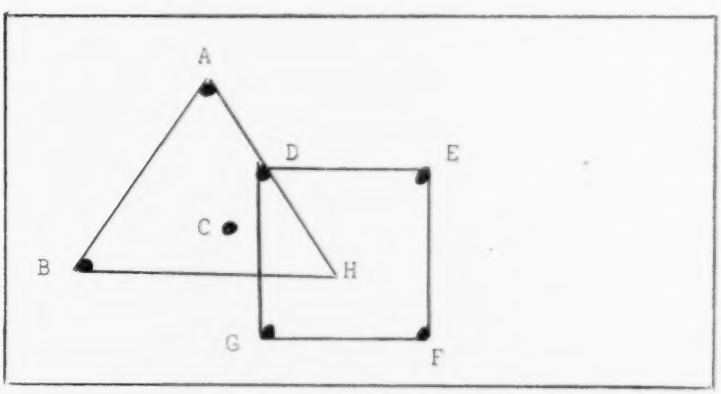


Figure 11

Have a pupil place a colored peg (point) inside the triangle, inside the square, outside the triangle, outside the square, inside the triangle and square, inside the triangle and outside the square. Have the pupil point to a peg (point) on both the square and triangle.

7. Label the pegs (points) with letters as shown. We may need to remind the children that the letters are used so we know which points we are talking about.

8. Play a guessing game as follows: Describe a point by its position and let the children guess the name (letter) of the point (see Figure 11).

   Example: I'm thinking of a point (peg) that is inside the triangle.

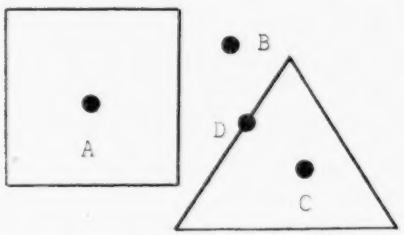
   Answer: Point C.

9. Using Practice Page 1 on page 40, have the children work the first exercise with you, and then let them complete the others by themselves.

   (a) This Practice Page is for further work with the terms inside, outside and on. If the preceding activities have been used, the pupils' memories need only be refreshed by going over the first part of the exercise. It is advisable to allow the pupil to read the instructions whenever possible. In doing the first exercise, you may call the student's attention to the use of the letters to name the dots (points). Have the pupils verbalize their answers and discuss their reasoning.

10. Practice Page 2 on page 41 is for further practice with the terms inside, outside, and on. If you have used the preceding activities, we need only refresh the pupils' memories by going over the first part of the exercise. It is advisable to allow the pupil to read the instructions whenever possible.

PRACTICE PAGE 1



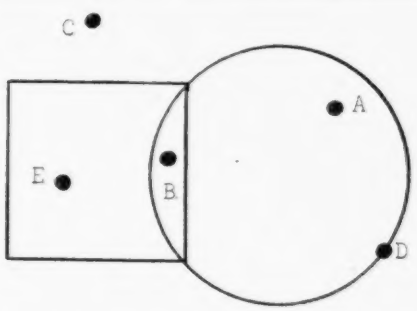
INSIDE THE SQUARE _____

INSIDE THE TRIANGLE _____

ON THE TRIANGLE _____

E

OUTSIDE THE SQUARE AND OUTSIDE THE TRIANGLE _____



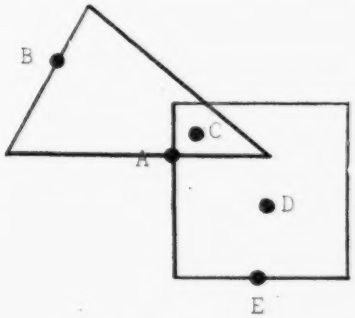
INSIDE THE CIRCLE _____

INSIDE THE TRIANGLE _____

OUTSIDE THE SQUARE _____

ON THE SQUARE. _____

INSIDE THE SQUARE AND INSIDE THE CIRCLE _____



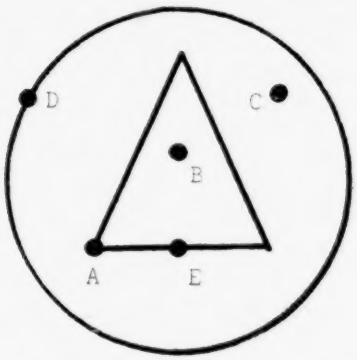
ON THE SQUARE _____

OUTSIDE THE SQUARE _____

ON THE TRIANGLE AND
ON THE SQUARE _____

INSIDE THE TRIANGLE AND
INSIDE THE SQUARE _____

PRACTICE PAGE 2

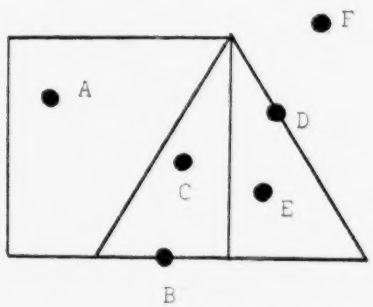


INSIDE THE TRIANGLE _____

INSIDE THE CIRCLE _____

ON THE TRIANGLE _____

OUTSIDE THE TRIANGLE AND INSIDE THE CIRLCE _____



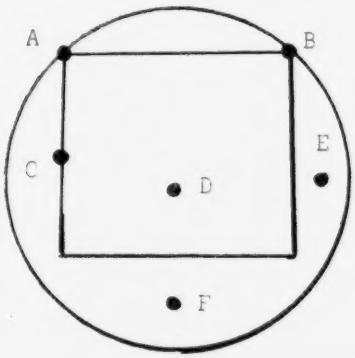
OUTSIDE THE SQUARE _____

ON THE TRIANGLE _____

OUTSIDE THE TRIANGLE _____

ON THE SQUARE AND ON THE TRIANGLE _____

INSIDE THE SQUARE AND INSIDE THE TRIANGLE _____



ON THE CIRCLE _____

ON THE SQUARE _____

ON THE CIRCLE AND ON THE SQUARE _____

INSIDE THE CIRCLE _____

INSIDE THE CIRCLE AND OUTSIDE THE SQUARE _____

ACTIVITY SEQUENCE V

Objectives:

1. To introduce measurement as a means of comparison.
2. To provide a sufficient number of experiences so as to develop a need for and, in turn, an understanding of a standard unit of measure.
3. To develop an intuitive understanding of the various standard units of measure, such as the inch, the foot, and the yard.
4. To present activities that are conducive to the learning of an estimation skill.
5. To illustrate through appropriate choice of measuring units that measurement of length is linear.
6. To demonstrate the need to be practical about selecting different units of measure.

Activity 1:

This activity relies on the child's natural desire to compare, for example: "I'm bigger than James," or "Michelle is shorter than Billy." This comparison is a basic form of measurement where the unit of measure is the individual himself or any nearby object. Such activities will allow the child to become familiar with such comparative terms as shorter, longer, etc.

Materials:

Yarn, string, pencils, rope, etc.

Arrangement:

Class observes while individuals perform certain tasks.

Procedure:

Select two students from the class and ask which is the taller. Once the answer has been given, ask pupils how they know their answer is correct. We would expect some kind of comparison to be made. The students will probably need some help in stating that they compared one student to the other. However, we must be sure to allow the students ample opportunity to express their thoughts. Once this comparison and discussion have taken place, similar activities should be used with objects, such as pencils, chalk, strings of different length, etc.

Activity 2:

We are attempting to show the child that in measurement we must always begin with a common origin, and that changing the position of an object does not alter its length; it may, however, change its appearance. A young child should not be told this; he must learn such a concept by experiencing many comparative activities.

Materials:

Yarn, feltboard, rope or string.

Arrangement:

The class plays the game as a group.

Procedure:

1. Prepare several lengths of yarn, rope, or string, one of which is as long as your desk or some other convenient object (see Figure 12 on following page). Place the yarn or string on the bulletin board and ask the students if they think any of the pieces on the board are as long

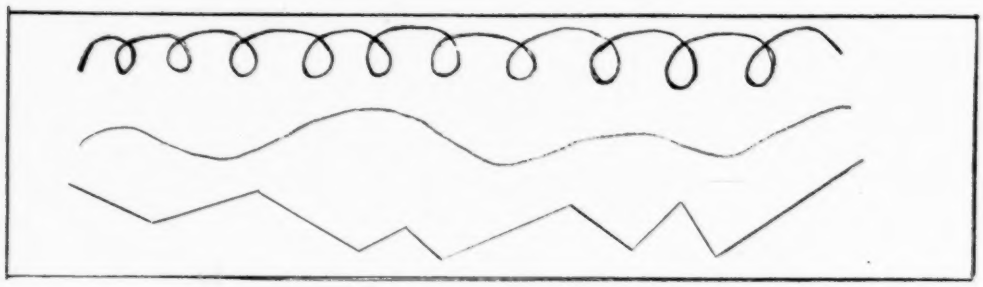


Figure 12

as the desk (or other convenient object you have selected). They should be allowed to guess, and as each child answers, his guess should be confirmed or shown false by taking the string he chose from the bulletin board and comparing it with the desk. Several wrong answers will most likely be given because of the way the pieces are arranged on the board. This could be discussed in terms of how we compare lengths. For example, we must stretch out the rope or string first (see figure 13 below).

2. Again, set up a bulletin board similar to the above (see Figure 12). However, this time the strings will represent the length or height of several different objects in the room. Ask the children to identify the rope or yarn that is as long as the cabinet or some other object. Each guess should be verified by actual comparison.

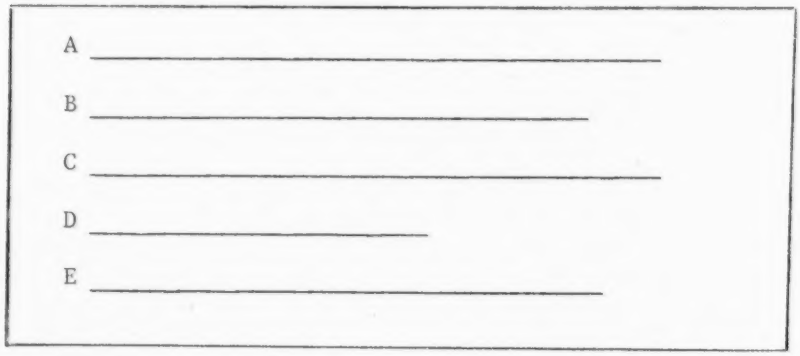


Figure 13

Activity 3:

This activity is a continuation of the preceding comparison activities, with one addition. We are now introducing measurement by a secondary comparison; that is, we no longer compare objects with each other, but by using a representation of their length. This concept is far more difficult than it appears and should not be treated lightly.

Arrangement:

Class observes, selected individuals performing assigned task.

Procedure:

1. Give each child a piece of string or strip of paper and ask him to cut it the same length as some object in the room, such as a book, pencil, etc. Have several pupils show their string or strip, and ask how they know the length is correct.
2. Have two children show how tall (measure) a third child is by using a piece of rope. Then, help the students tape the length of rope to the board and put the student's name by it. Repeat the process for a second child. After completing both measurements ask the rest of the class to tell by looking at the board who is the taller (see Figure 14).

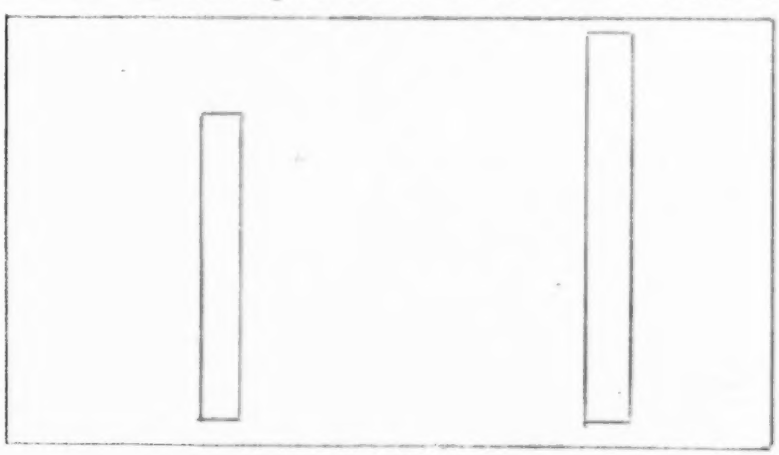


Figure 14

3. Send one child into the hall and ask another to step to the front of the room. Now ask the class who is the taller, the child outside or the one in front of the class. We are trying to have the children recognize the need for measuring each child by some means other than the actual physical side-by-side comparison; that is, a measurement strategy similar to that used in paragraph (2).

4. After completing the above activities, the children may find it interesting to represent all their heights on one of the chalk or bulletin boards. This can be done by having the children measure each other with string or yarn, cut off the appropriate length, and place it on the board with the name under it (Figure 15).

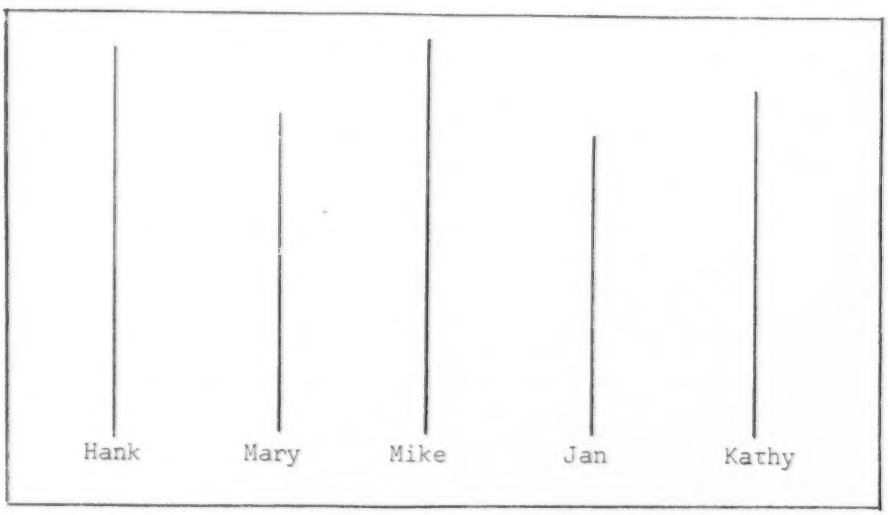


Figure 15

This type of activity can be thought of as the first readiness activity for graphing. You may want to make a new graph later in the year to show change, so the original graph can be stored for later use.

Activity 4:

With this activity we hope to develop the notion that many different objects can serve as a unit of measure. We have listed only a few items that may be used. The teacher should feel free to use any additional units of measure she finds appropriate.

Materials:

Rope, string, paper clips, chalk, etc.

Arrangement:

Class observes as selected students or small groups perform.

Procedure:

1. Have as many as possible of the above mentioned items available on a table for the childrens' use in measuring. A record should be kept for each of the following five activities.
   (a) List each object and its length, for example, "The desk is 5 ropes long."
   (b) Select a child to help measure the length of your desk with a piece of rope. Ask, "How long is the desk?" Then measure the desk by placing the rope along the desk and having the child mark the end of the rope with his finger. Move the end of the rope to his finger and continue as before. Of course, your answer will be approximate.
   (c) Pick other children to help in measuring different objects with different units of measure, until you think they have mastered the mechanics of measurement.

(d) Now, have the students measure the same object with different units of measure. Record the answers as suggested and then discuss the reason for the difference in the answers. For example, "The desk is 4 ropes long, but it is 7 strings long."

(e) After using many different examples similar to the two above, we hope to conclude that in order to have measurements which are understood by everyone, we should use the same unit of measure or we should measure with the same thing.

Activity 5:

This activity not only continues to develop the last concept in Activity 4, but it is a readiness activity for the standard units of measure called inch, foot, and yard.

Materials:

None

Procedure:

Ask the class how far it is across the room. Do not immediately suggest a way to measure it, but let the children discuss how they would want to do it. After talking about several ways, you can suggest the number of steps as one means of measuring. Have a child tell how many steps he thinks it would be, then have him try it. Once he obtains a number, count the number of steps it takes you to walk across the room (which will be different from the student's). At this point, we can ask why we got different answers. Next, measure the same distance by placing one shoe immediately in front of the other (see Figure 16 on the following page).

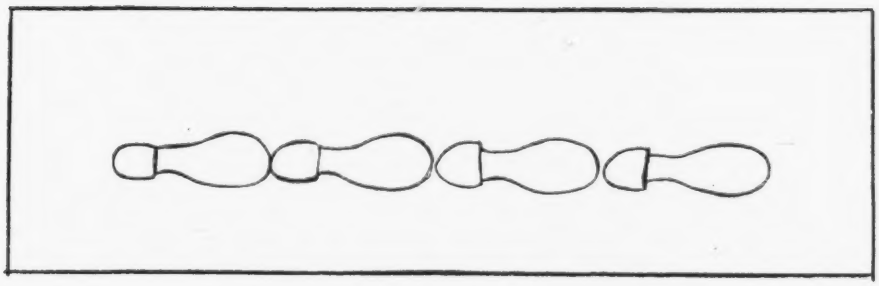


Figure 16

Have the students count the number of times you use your shoe and then select a student to do the same type of measurement. Again, we should discuss why we get a different number of steps (keep a record of each measurement on the board).

Activity 6:

Once the child comprehends why there are fewer or more steps (Activity 5) when different people are measuring, we can introduce the elf shoe, the human shoe, and the giant shoe as units of measure (Figure 18). These shoes are one inch, one foot, and one yard in length; however, the child is not to be told this. The objective of this development is to show that the name of the unit is arbitrary and may be called anything we wish. Also, we want to introduce the notion of one measure being part of another. For instance, one human shoe is as long as 12 elf shoes.

Materials:

Elf shoe (1-inch long), human shoe (1-foot long), giant shoe (1-yard long), and various other measuring units. Cut these shoe-shapes from poster board and draw on them so they look like the shoes on page 51.

Arrangement:

Introduce in large groups and then use groups of 4-6 children.

Procedure:

Before proceeding, review a few measurements using a child's shoe (see Activity 5) and the teacher's shoe

Ask the children how long they think a giant's shoe would be. After several guesses, bring out the cutout of the shoe. Discuss its length and whether or not a man's shoe would be shorter. Then bring out the cutout of a man's shoe. Similarly, introduce the elf shoe. (Please feel free to make up any appropriate story with the children to introduce these items. The only necessary goal is that they observe the relative lengths).

After the introduction of the above units, select objects to be measured and help the children measure the objects. You may wish to put down strips of tape on the floor to be measured.

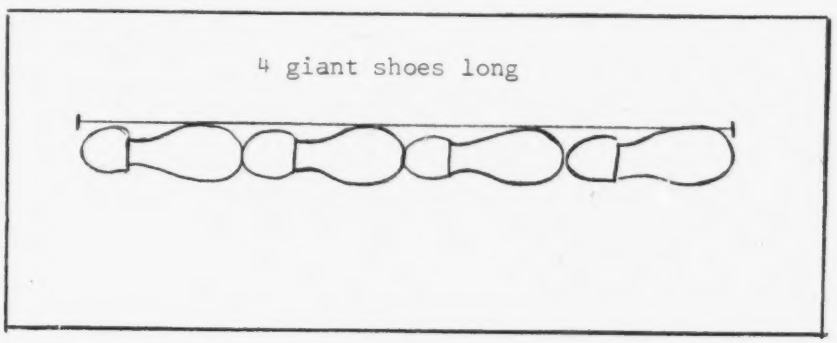


Figure 17

Give each child as much practice as possible with these measurement tasks before proceeding to any new material.

Activity 7:

It will immediately become apparent that not everyone who wishes to measure a distance will have a set of shoes, so something else must be done. In this activity, we will substitute lengths of wooden lattice for the cardboard shoes. We will pretend that the lattice is the same as one of the shoes; that is, it is the same length.

Materials:

Wooden lattice of the same length as cutouts of elf, giant, and human shoes.

Arrangement:

Introduce in large group and then use groups of 4-6 children.

Procedure:

Display the set of shoes and the lengths of lattice as follows:

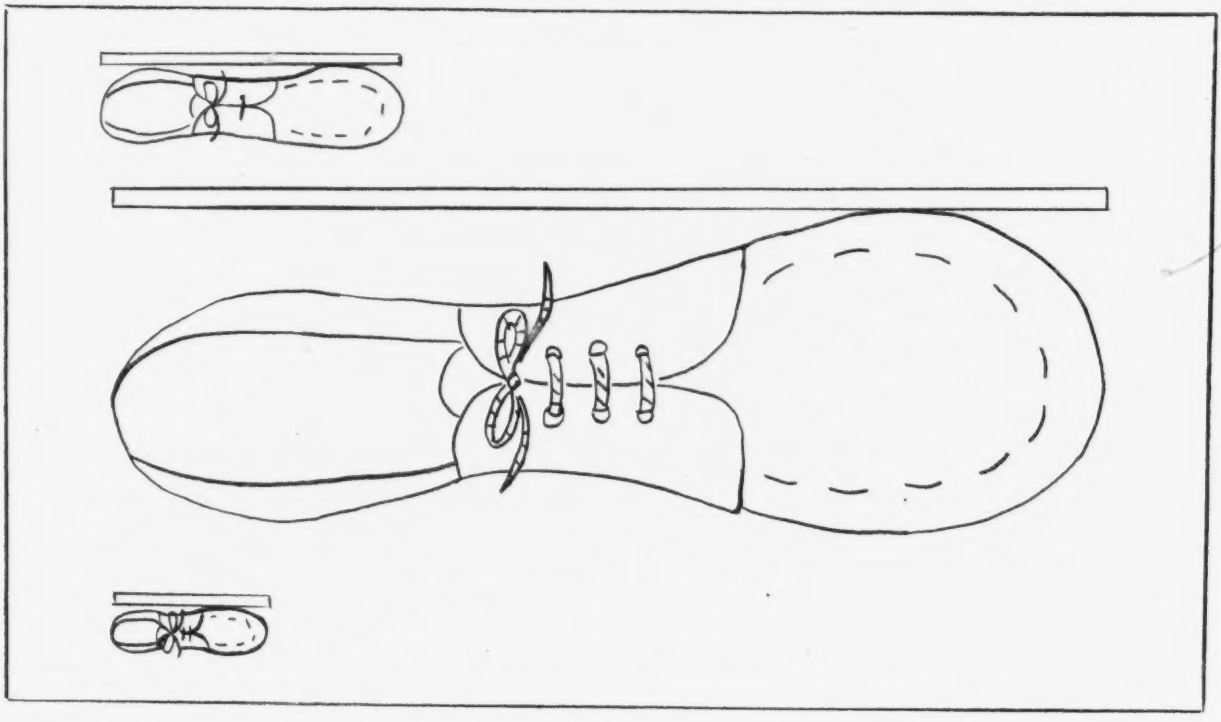


Figure 18

Have a child compare the giant shoe and the lattice above it. We, of course, want all to see that they are the same length. The same comparison should be made with the other shoes and pieces of lattice. This comparison can also be illustrated by first measuring some object with the shoe and observing that both measurements result in the same answer. After completing several such measurements, we could discuss calling the lattice by the names: giant shoe, human shoe, and elf shoe, explaining that this would allow everyone to have a set of shoes.

At this time the 1-yard stick becomes the giant shoe, the 1-foot stick the human shoe, and the 1-inch stick is the elf shoe.

Activity 8:

This activity is aimed at developing a reasonable selection of a unit of measure; that is, the child would not measure the length of the hall with an elf-shoe unit nor a book with a giant-shoe unit. Also, we will begin the development of estimation.

Materials:

Giant shoe, elf shoe, human shoe (now using the lattice form of units).

Arrangement:

Preferably small groups.

Procedure:

1. At the beginning of the activity, remind the children that we are to call the sticks by the giant shoe, elf shoe, and human shoe names. "We are doing this because each of us has a set of sticks which we call shoes and so it is easier to do our measurements." Have a student choose

an object he wants to measure. Then call on another student to measure the same object. However, before this second student can measure it, he must pick the unit of measure he wants to use (piece of lattice). Once he has measured the object and you and the class are satisfied he is correct, he may choose the next object to be measured. This activity can be continued as long as the student's interest will allow. While making these measurements with a giant shoe, we would ask, "Do you think it would take more or fewer human shoes to measure the object?"

2. Use the same unit of measure as in (1) above. Choose an object to be measured, and then have a student guess how long it is compared with one of the sticks. He should use the name of the stick in the guess. After the guess has been made, check his guess with the help of the rest of the class. Even if the guess is inaccurate it should not be termed wrong since it is only an estimate. Help the children to improve through further practice. Always check estimates by measuring!

3. After completing the activities above, choose one object that has been measured with different units of measure and have two children (one at a time) measure it with the same unit of measure. After each has given his answer (hopefully the same), discuss this with the class.

NOTE: Each of the above activities is merely representative of the sequence that we expect a child to follow. But this list should not be considered mandatory; on the contrary, the teacher will make this unit a success by adding or deleting activities according to her judgement.

Activity 9:

This and other similar activities are to be used to help the child understand the use of multiple units of measure. For instance, we may say that the

desk is two giant shoes and one human shoe long.

Materials:

Three pieces of lattice (giant, elf and human shoe size) for each child; cutouts of one dog, one doghouse, one bowl and a rope.

Arrangement:

Small group if possible.

Procedure:

Using the three sticks (shoes) present the following problem:

Spot is tied with a rope to his doghouse. But, as you can see, he can not reach his dinner in the bowl (Figure 19), "Can you give him a piece of rope long enough so he can reach his bowl?" (More than enough rope should be provided.)

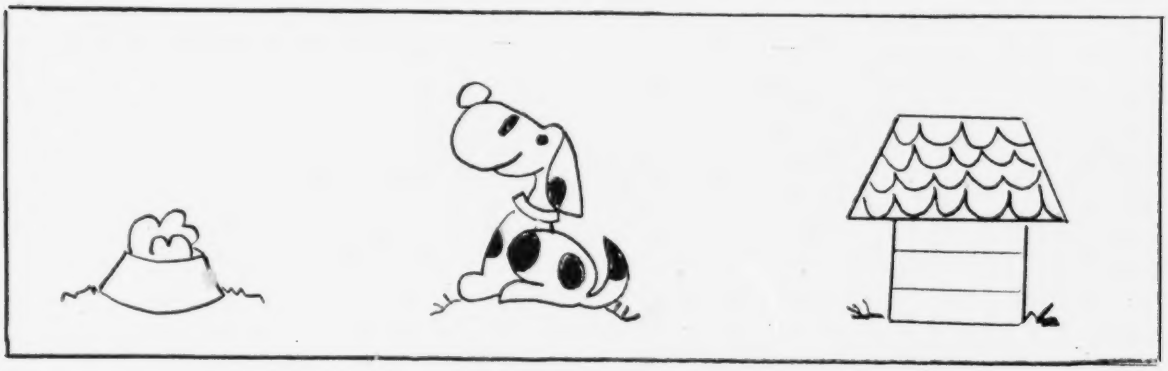


Figure 19

The first answer will most likely be found by picking up all the rope, comparing it to what is needed, and then cutting it. Next ask the children what they would do if they didn't have the rope and had to go down to a store and buy it. Refer to the three sticks and ask if they could use those to measure what they needed. It may be difficult to get any practical results at first, but we would hope eventually to arrive at an answer that would be approximately correct. The teacher may be able to elicit an answer such as "The rope will have to be two giant shoes, a human shoe, and an elf shoe long."

ACTIVITY SEQUENCE VI

Objectives:

1. To introduce linear measurement as a means of finding perimeters.
2. To provide further experiences in the use of previously developed units of measure.
3. Each child is to learn in an oral and informal way the relation concept of betweeness and the use of the mathematical relations greater than and less than.
4. To reinforce the term path as a means of discussing a given line segment or segments.

Activity 1:

This activity is to function as a continuation of the development of the concept of standard unit. Therefore, we must be sure to use a positive approach to incorrect or partially correct answers.

Materials:

Geoboards, worksheets, rubber bands.

Arrangement:

Tc insure that each child is introduced to the concept correctly, it will be advisable to have groups of 4-6 children for this activity. Also, since the activity has several parts, it is possible that it may not be completed in a single class period.

Procedure:

1. Construct a square on your geoboard and ask how long each side is. Let the answers vary as far as the name of the measure is concerned. It might be so many spaces, nails, etc.
2. Construct a rectangle and proceed as above, but guiding the children into using the term unit, that is, "it is 6 units long".
3. Construct paths on your geoboard similar to the following:

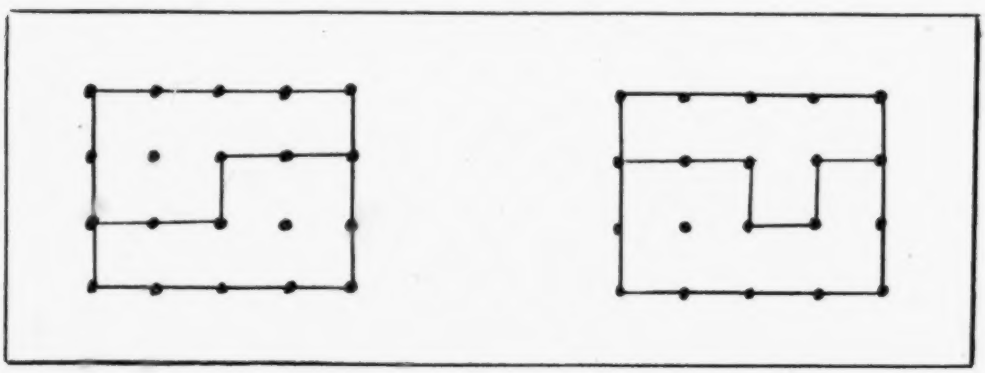
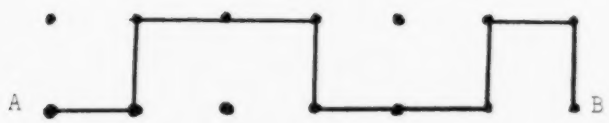


Figure 20

Ask the children how many units long are each of the paths.

4. Present the children with the work sheet (Exercise I) on page 57. Work the first two examples with the class and then let them finish alone.
5. Using Activity 1 (Exercise II) on page 58, ask the children how many units it is from A to B? How many units from B to C? How many units from C to D? How many units from D to A? Then ask how many units is it if we go all the way around, starting at A and ending at A. Record this answer on the line numbered 1 below the figures. Do the same for 2 and let the children work 3 and 4 by themselves.

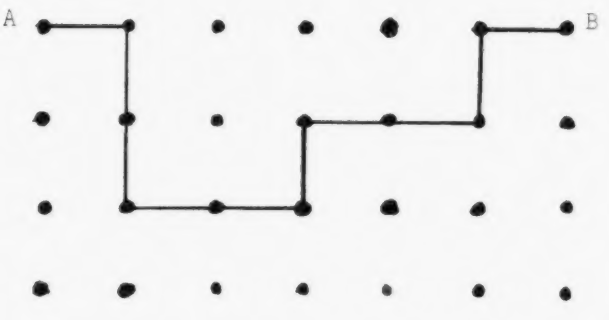
Exercise I



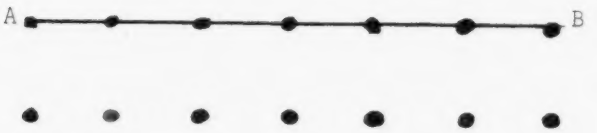
1. _____ Units

2. _____ Units

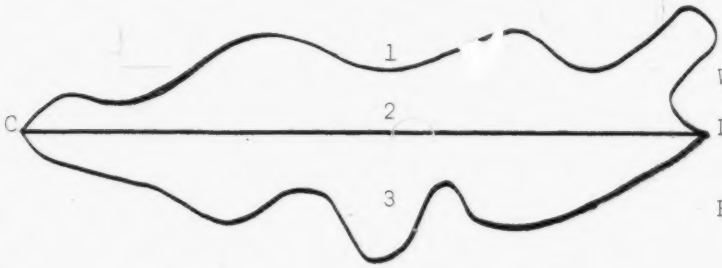
3. _____ Units



4. _____ Units



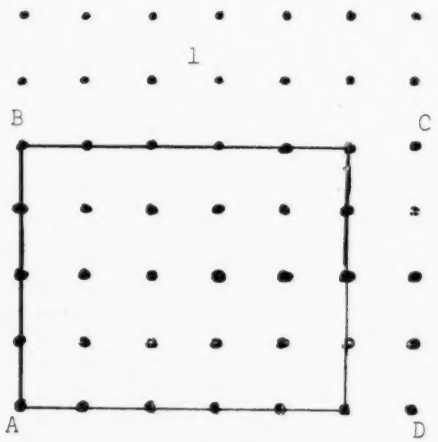
5. _____ Units

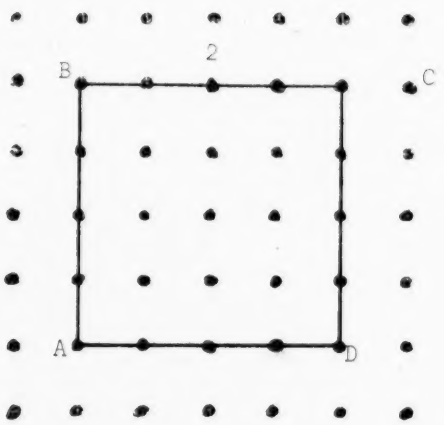


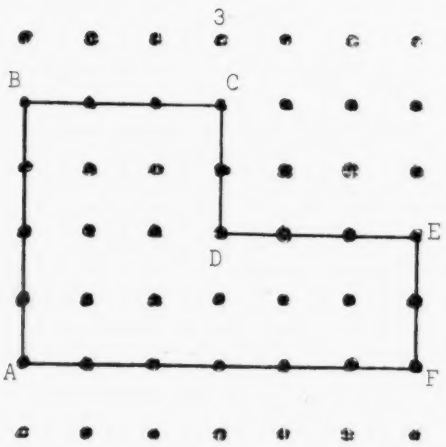
6. Which path is the shortest?

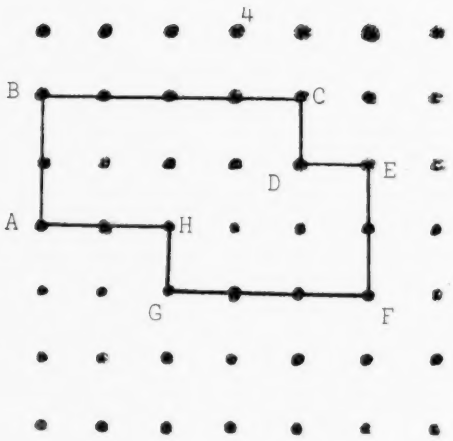
Path 1 Path 2 Path 3

EXERCISE II









1. _____ Units

2. _____ Units

3. _____ Units

4. _____ Units

Activity 2:

In this activity some of the children will select from a set of objects an appropriate measure to find out the distance around objects. The children will be asked what the total distance is around the objects rather than the length of each side.

Materials:

Objects for Measuring: clothes pins, paper clips, sticks of varying lengths, string.

Objects to Measure: clothes hanger, napkins, wallet, kite, checkers, etc.

Arrangement:

The class works as a whole by having different children select what is the best unit of measure for them.

Procedure:

From the groups of objects (paper clips, clothes pins, sticks of varying lengths) have the children select the object they would perfer to use for measuring the clothes hanger, napkin, kite, etc. Encourage the children to use paper clips, clothes pins, and sticks on the same figure. Most children will soon recognize the need for a fixed standard unit of measure. The teacher should encourage the children to express in discussion the problems they would have using many different units and the advantages of using a standard unit.

Activity 3:

We can now refer back to the three units of measure that have already been established (elf, human, and giant shoes). Each child will be using the sticks they named elf shoe, human shoe, and giant shoe.

Materials:

Large cardboard cutouts of familiar objects, such as a flag, boat, etc.

Arrangement:

Every effort should be made to work in small groups during this activity.

Procedure:

Have the children select a cardboard cutout to be measured by their group. The object is then traced with chalk on the floor, after which at least two children and the teacher should use one of the "shoe" units to find the perimeter. After using different units for the measurements, discuss the reason for the variation in the numbers found in the measurements. If, in measuring the distance around the figure, the total distance is between 9 and 10, 10 and 11, etc., discuss the concepts of greater than and less than. some of the children will also reason intuitively that when a larger unit of measure is used, the number of total units will be smaller. When a smaller unit of measure is used, the number of units will be larger when measuring the same figure. If the group is interested in this comparison, a class chart could be started by the teacher and children to record of measures in terms of units used.

NOTE: As the opportunity arises, introduce similar activities.

Activity 4:

Without observing a particular group of children, it is impossible to predict when they are ready to use the names yard, foot, and inch for the standard units of measure we normally use. Thus far, we have used many different units of measure as a means to develop the need for and an understanding of a standard unit of measure. We have used standard names for some measuring units

such as giant shoe, human shoe, and elf shoe, but we need names that everyone will understand. Hence, we must use names that are familiar to a great number of people; that is the yard, foot, and inch.

Materials:

Each child will need the sticks we have been calling giant shoe, human shoe, and elf shoe.

Arrangement:

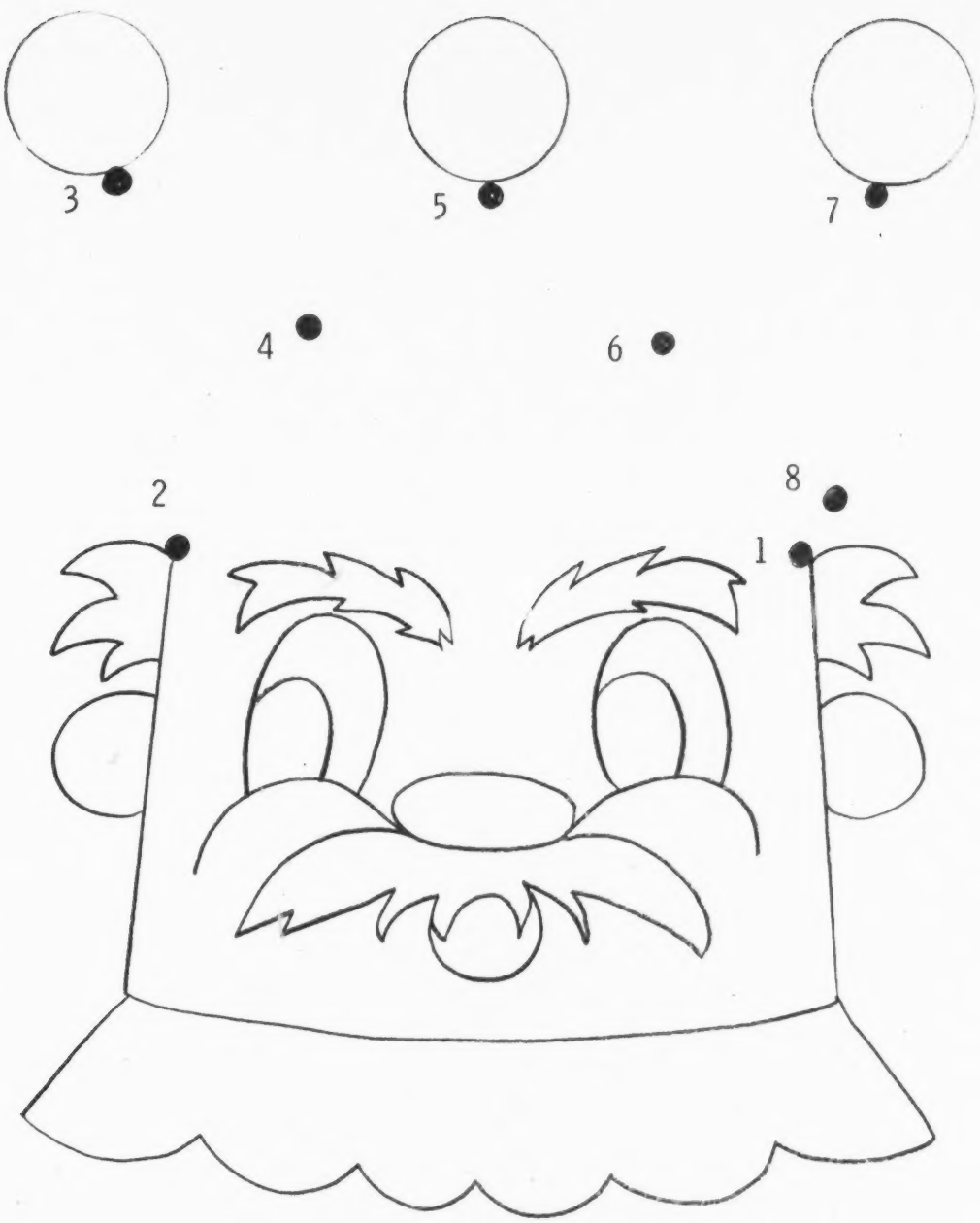
Class works as a whole.

Procedure:

Discuss the need for having words that many people understand and why we learn new words. Then tell the children we are going to rename our sticks so everyone (mother, father, etc.) will know what we are talking about. We will now call the elf shoe an inch, the human shoe a foot, and the giant shoe a yard. Explain and discuss the fact that the change of name does not change the way we use these measures, but that we now have new names for the sticks.

After discussing the new names, any measurement activity may be used to familiarize the children with them. No single exercise can be expected to do the job. You, as the teacher, should take advantage of every opportunity to engage the child in measurement tasks. Real problems that may arise in class requiring the use of measurement should be solved, whenever possible, by the children.

APPENDIX I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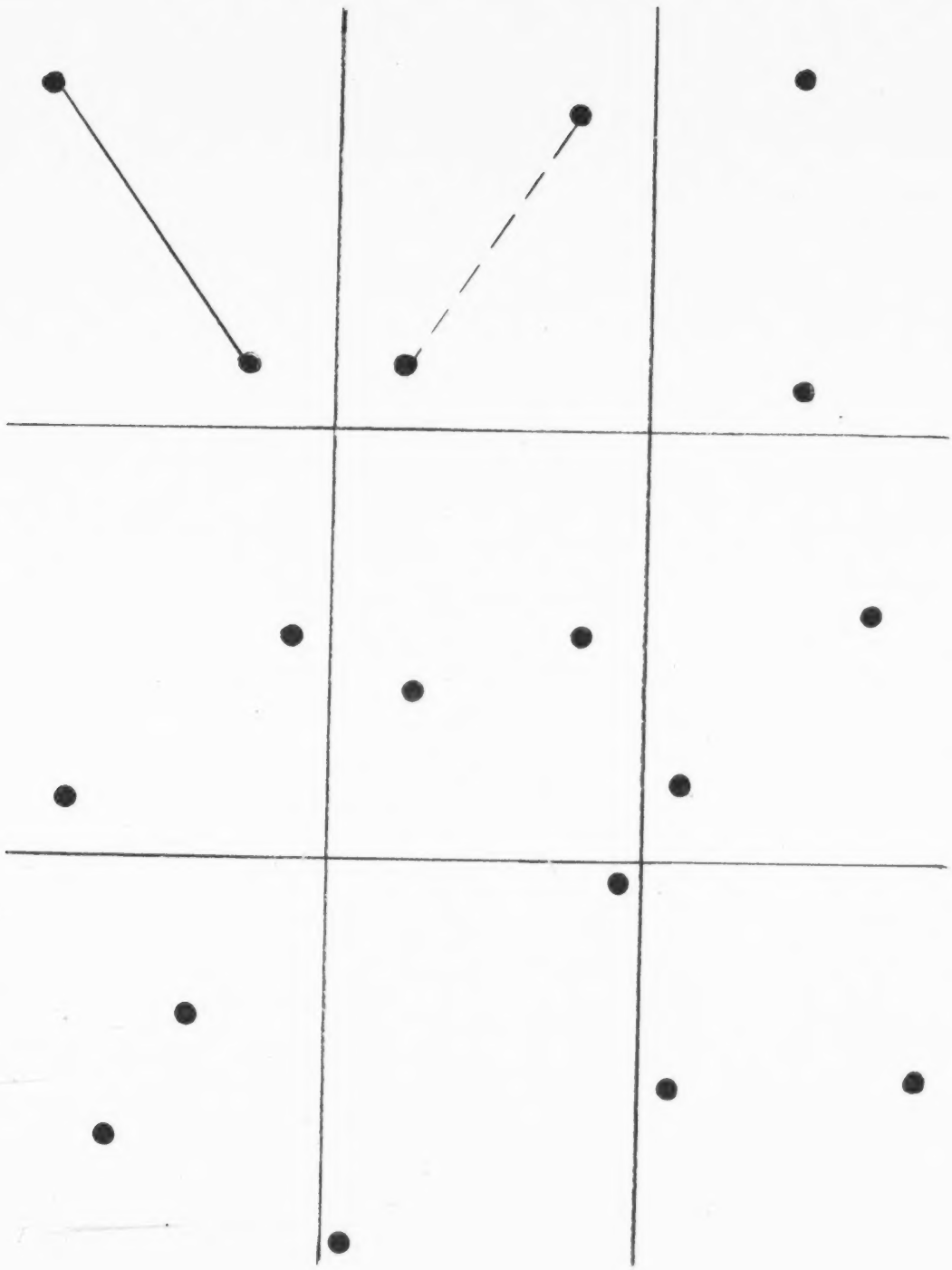

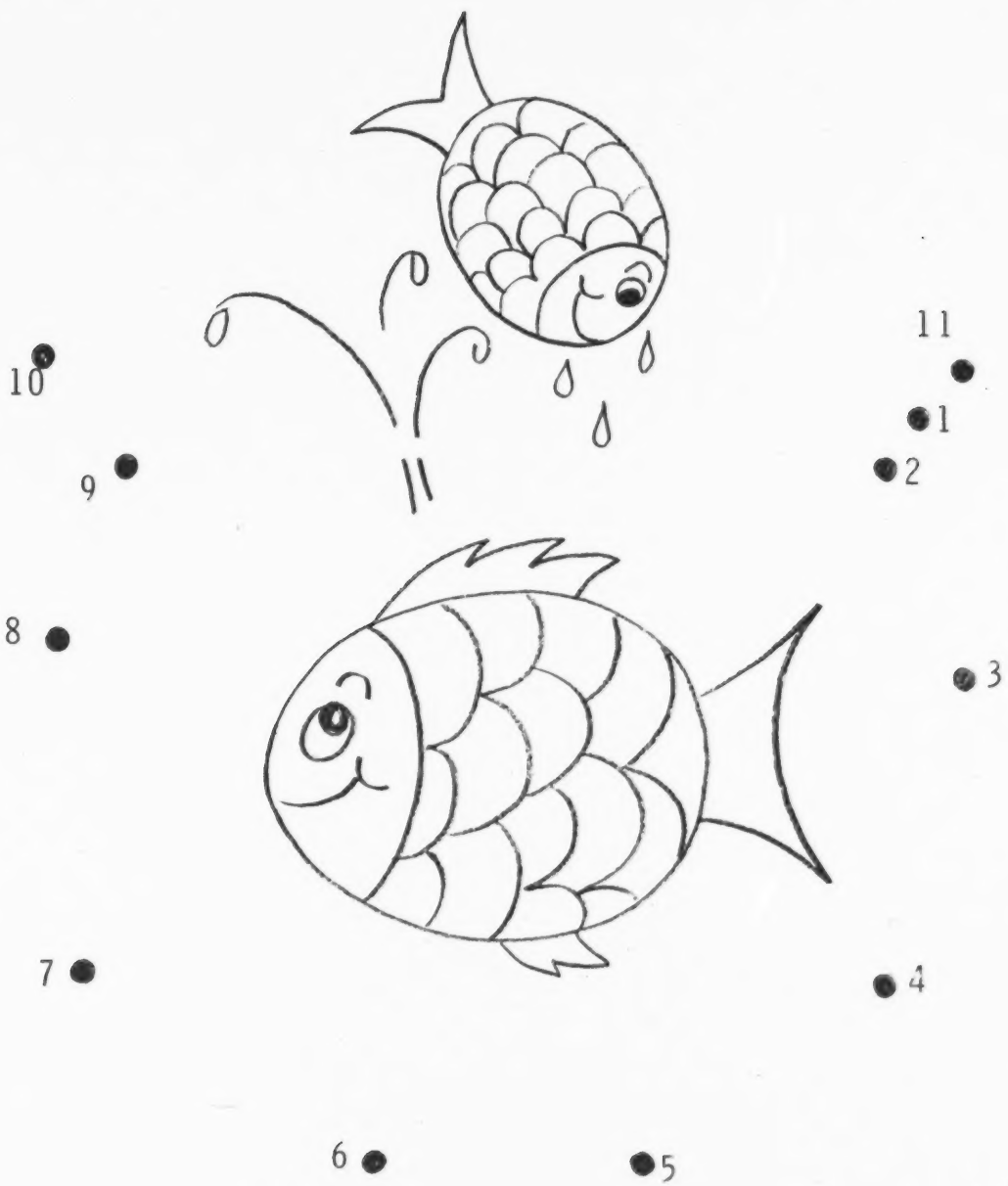
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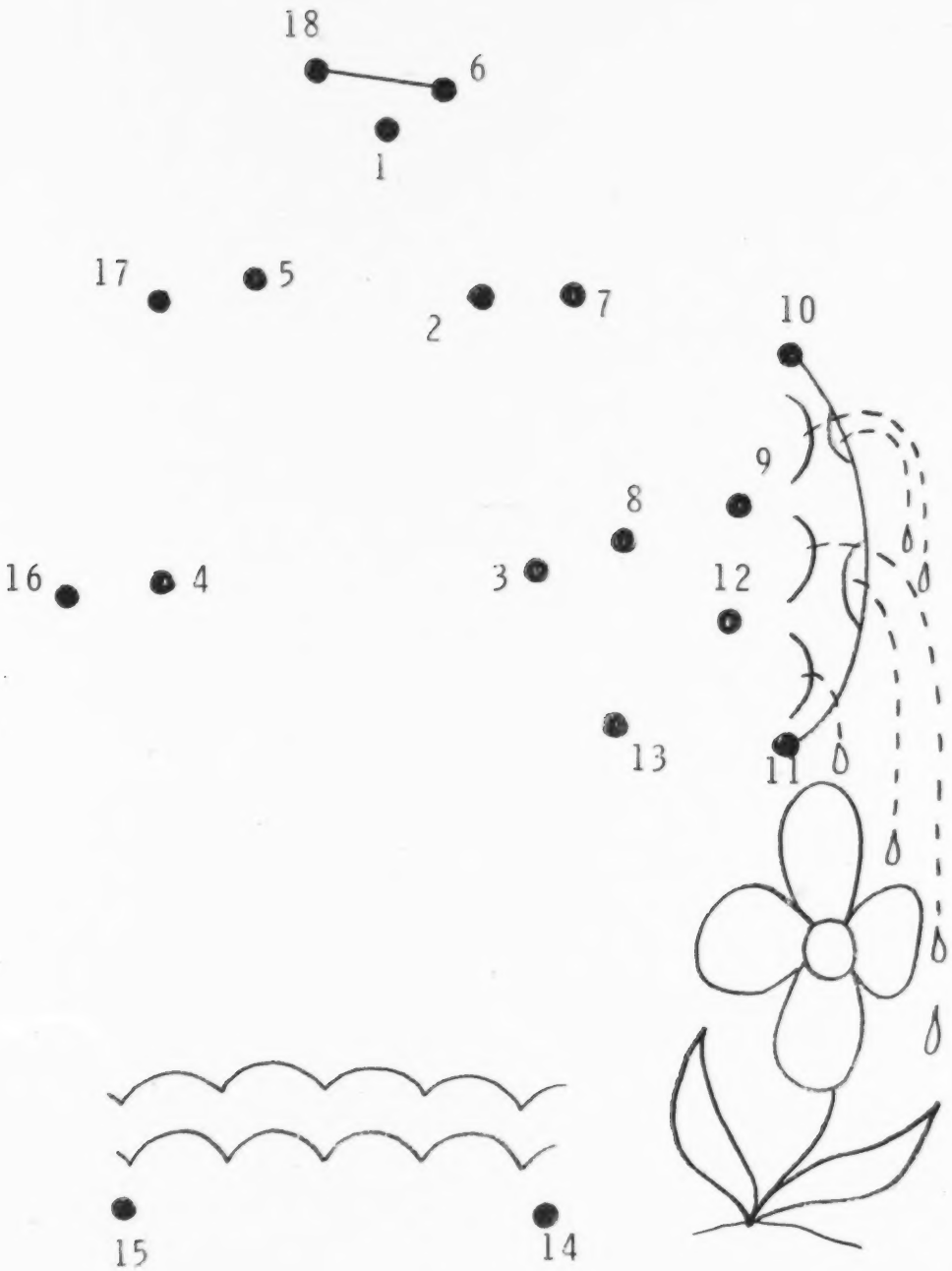
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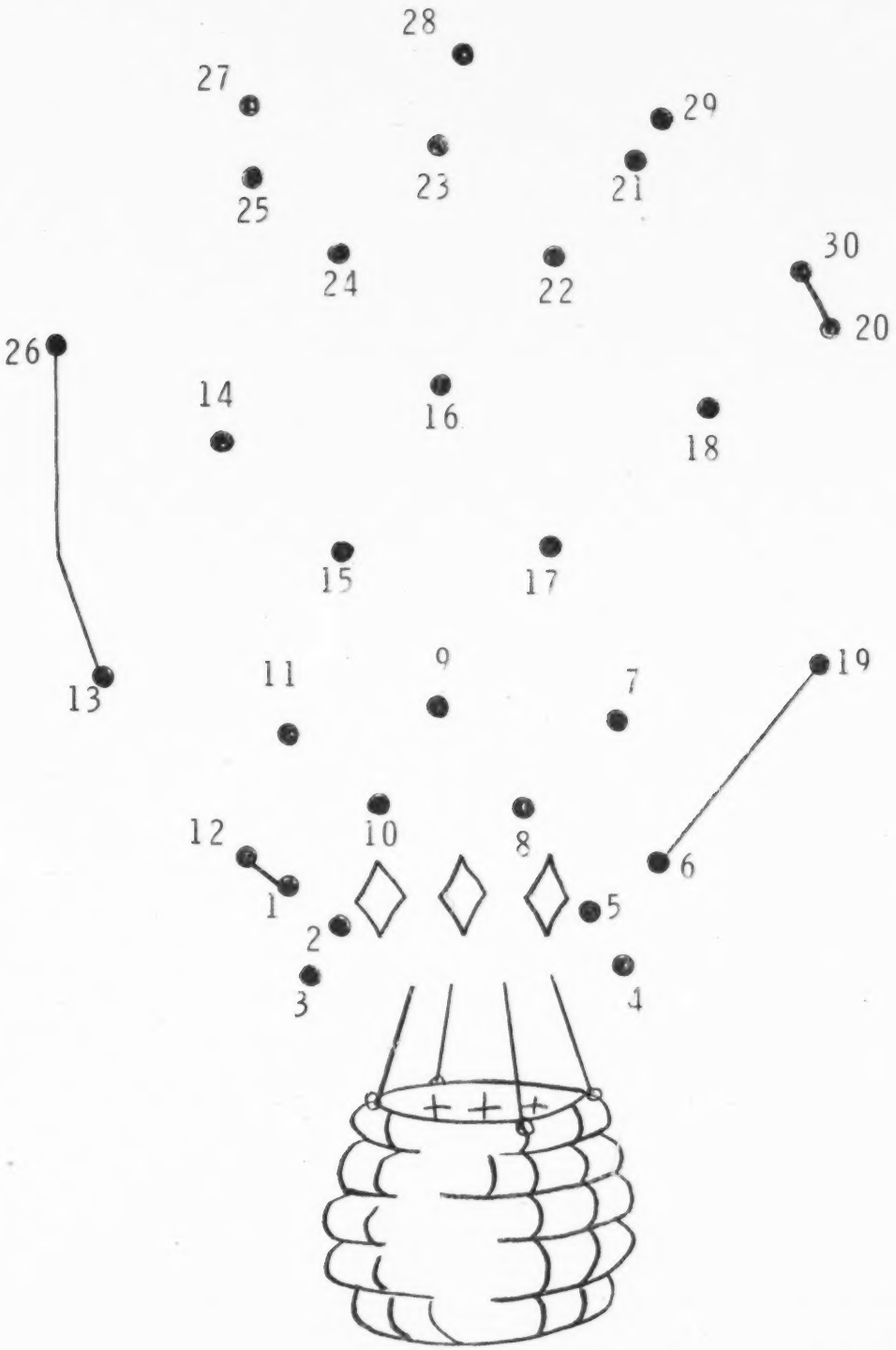
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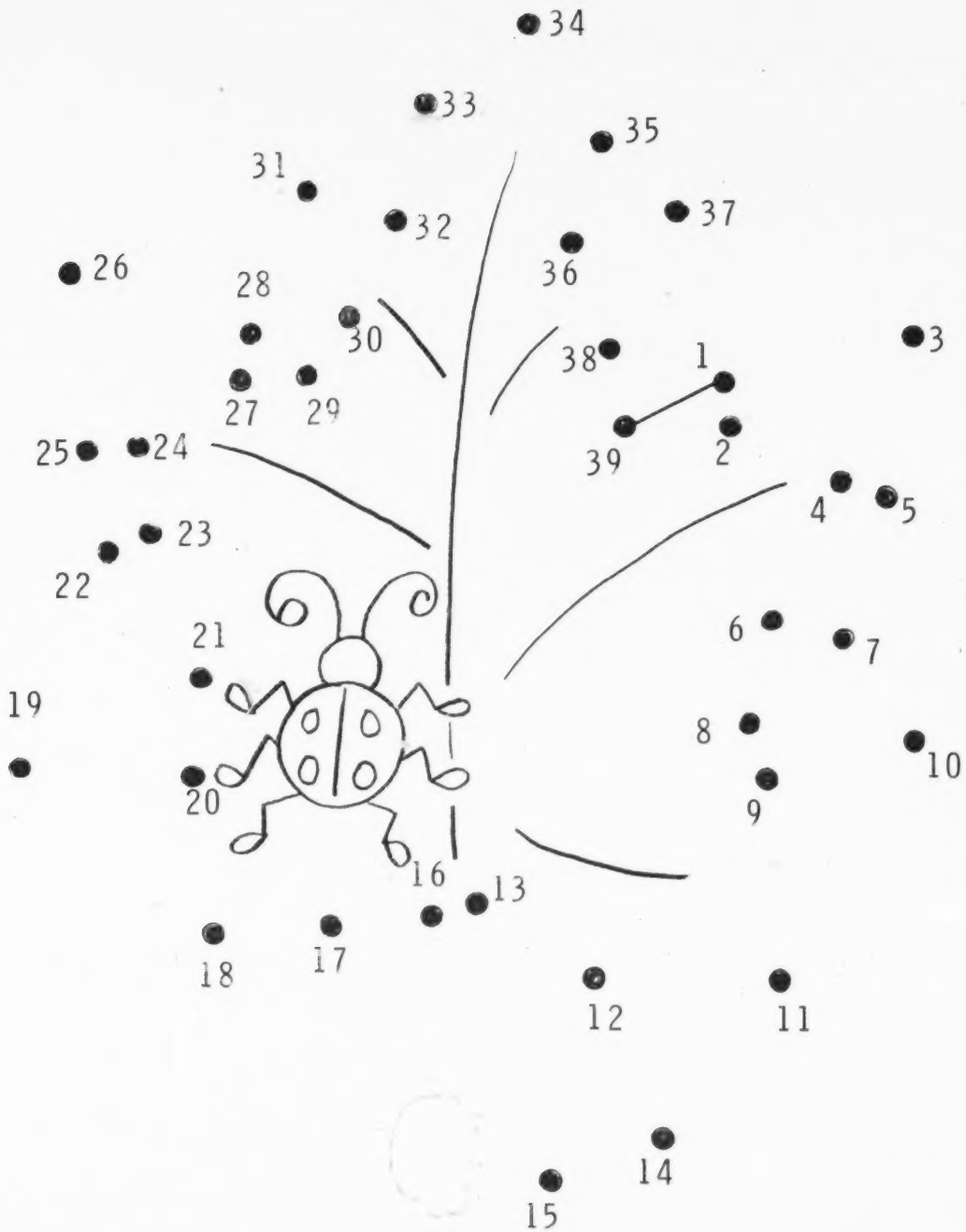
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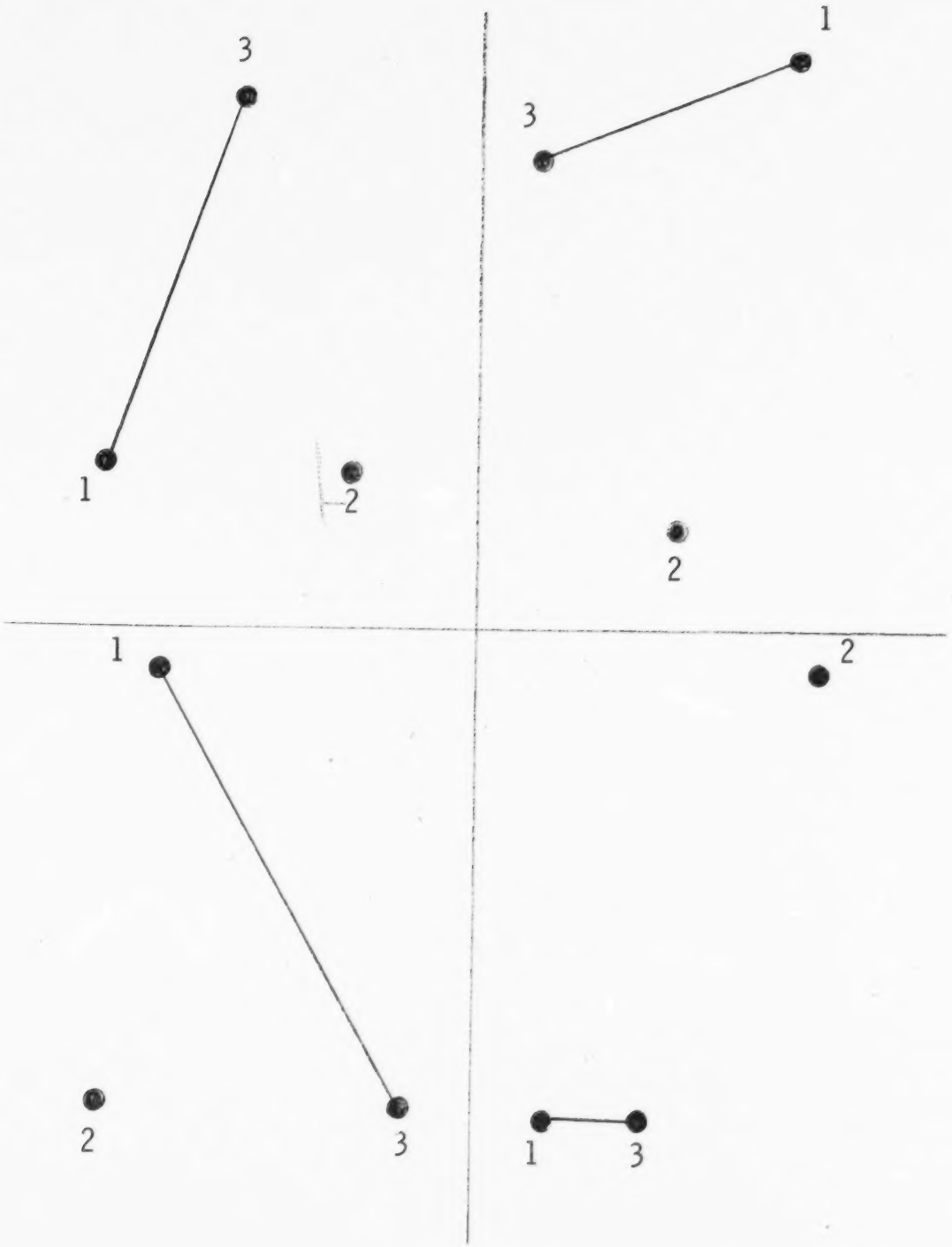
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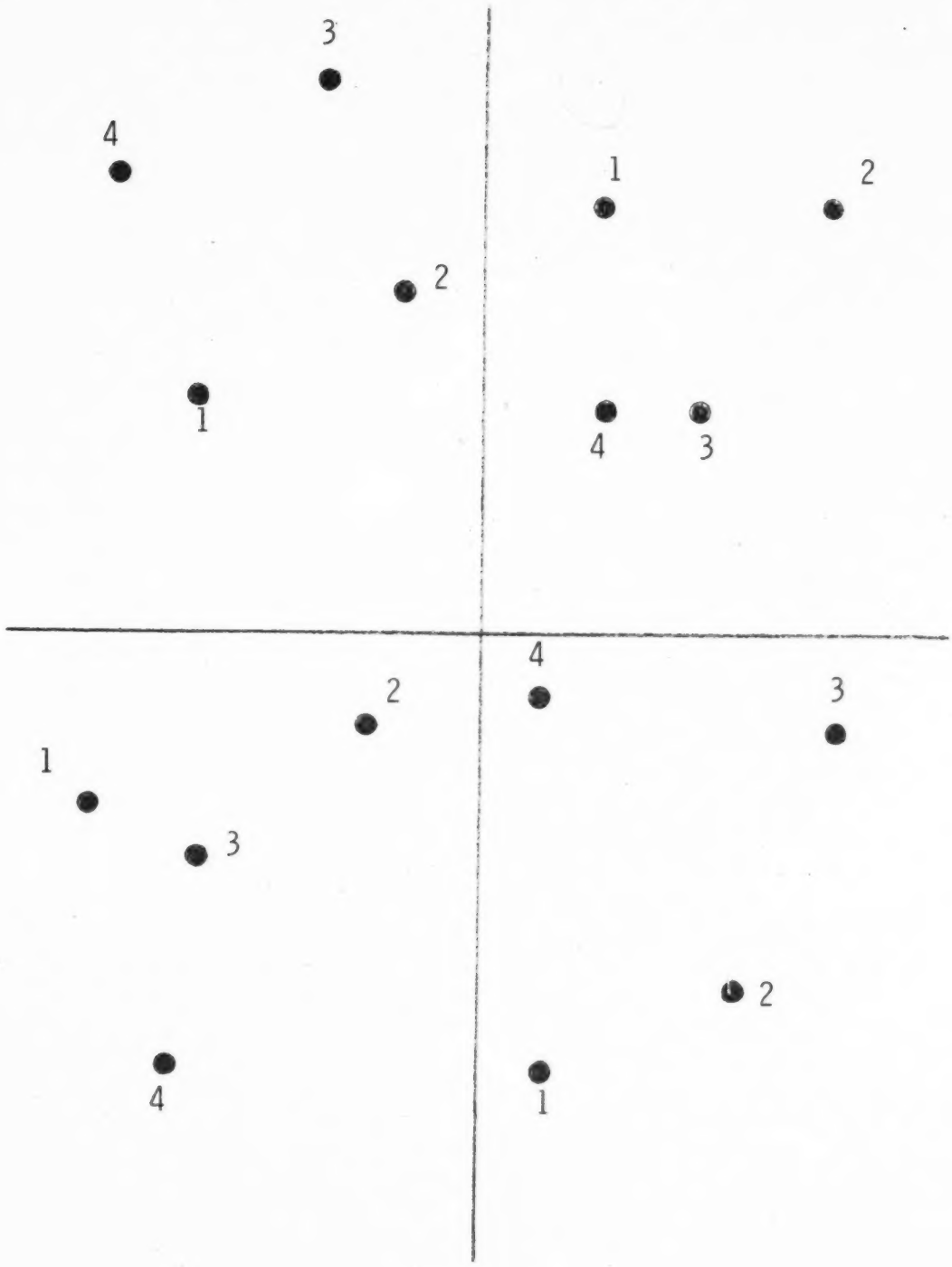
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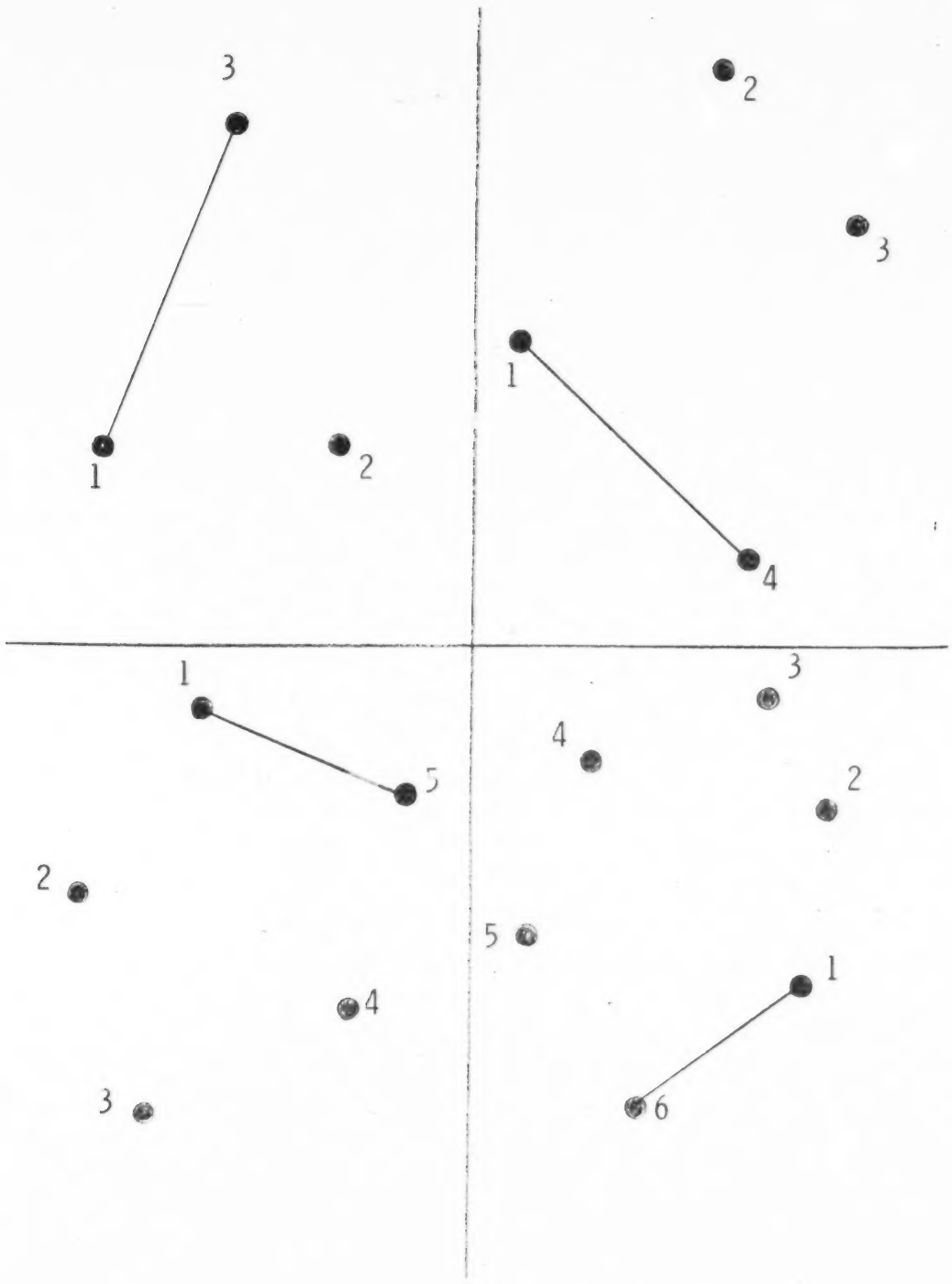
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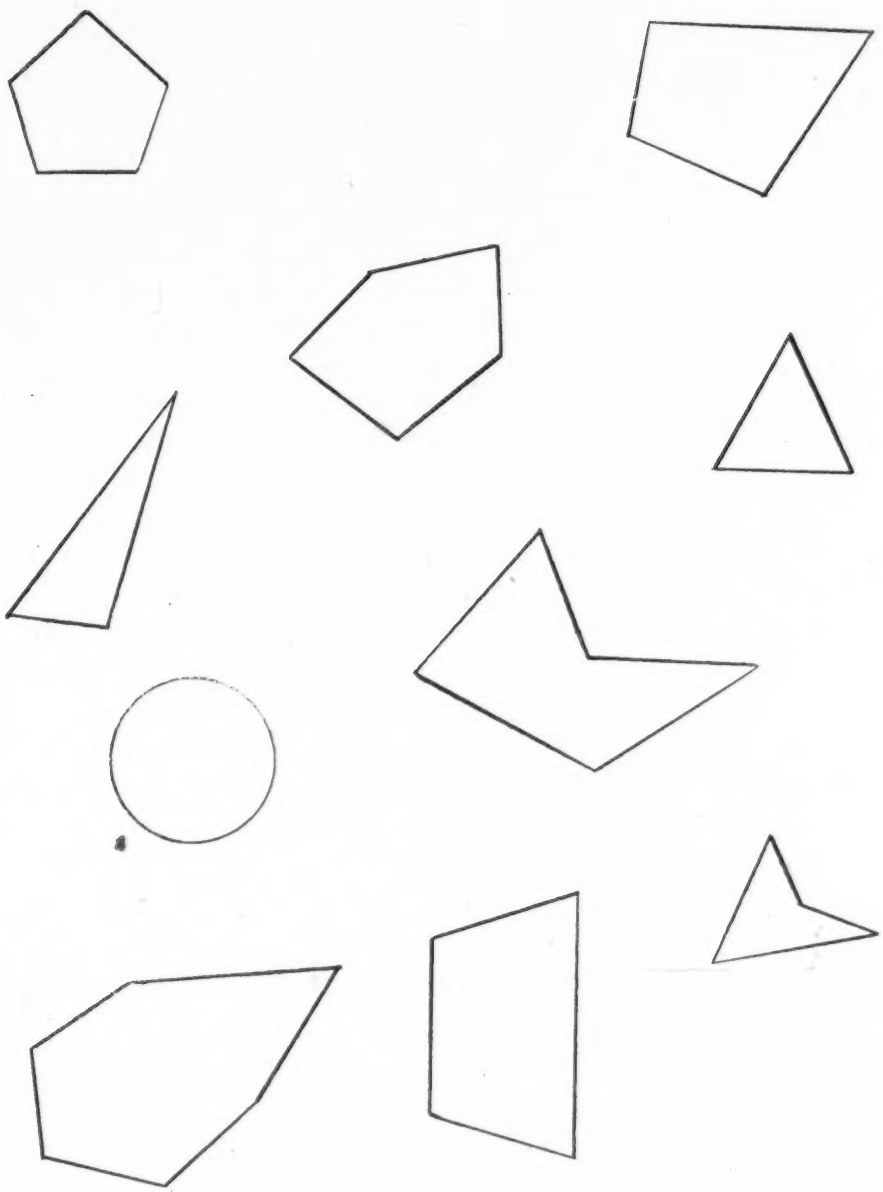

APPENDIX II

Shapes similar to these but enlarged approximately five times should be cut from posterboard for use in the activities described on page 31.